Math 120 - Winter 2015
Final Exam
March 14, 2015

Name: _____   Student ID no. : _____

Signature: _____   Section: _____

1	12	
2	13	
3	13	
4	9	
5	15	
6	13	
7	12	
8	13	
Total	100	

- This exam consists of EIGHT problems on NINE pages, including this cover sheet.

- Show all work for full credit.

- You may use a scientific calculator during this exam. Graphing calculators are not permitted. Other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.

- You do not need to simplify your answers.

- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.

- If you write on the back of the page, please indicate that you have done so!

- You may use one hand-written double-sided 8.5″ by 11″ page of notes.

- You have 170 minutes to complete the exam.

○

1. **[12 points]** In the following figure (not drawn to scale), find x.
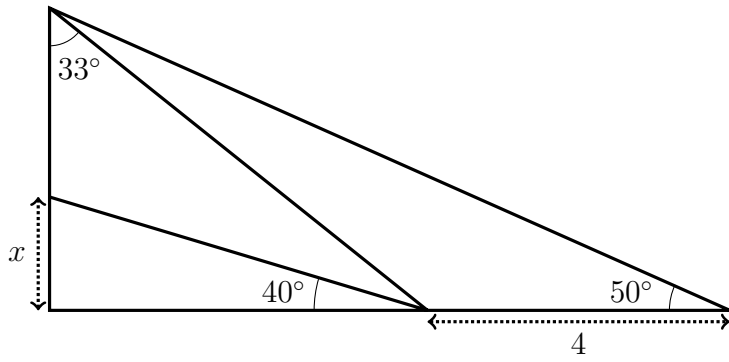
2. The number of trees in Treeattle grows exponentially.

   Treeattle had 600 trees in the year 2008, and 1100 trees in the year 2015.

   (a) **[4 points]** Write a function $f(x)$ for the number of trees in Treeattle, x years after the year 2000.

   (b) **[6 points]** Compute $f^{-1}(x)$, the inverse of the function you found in part (a).

   (c) **[3 points]** When will there be 4000 trees in Treeattle? Round your answer to the nearest year.

3. (a) **[3 points]** Write a function $f(x)$ for an upper semicircle of radius 4 centered at $(6, 2)$, defined over the interval $2 \leq x \leq 10$.

(b) **[3 points]** Write a function $g(x)$ for the curve obtained by taking $f(x)$ from part (a), moving it 2 units to the left, and *then* scaling it horizontally by a factor of $1/2$.

(c) **[4 points]** Find the domain and range of $g(x)$.

(d) **[3 points]** Is $g(x)$ one-to-one? Explain, briefly.

4. **[9 points]** In the following configuration, wheels A and B are connected by a belt, as are wheels C and D. Wheels B and C are connected by an axle.
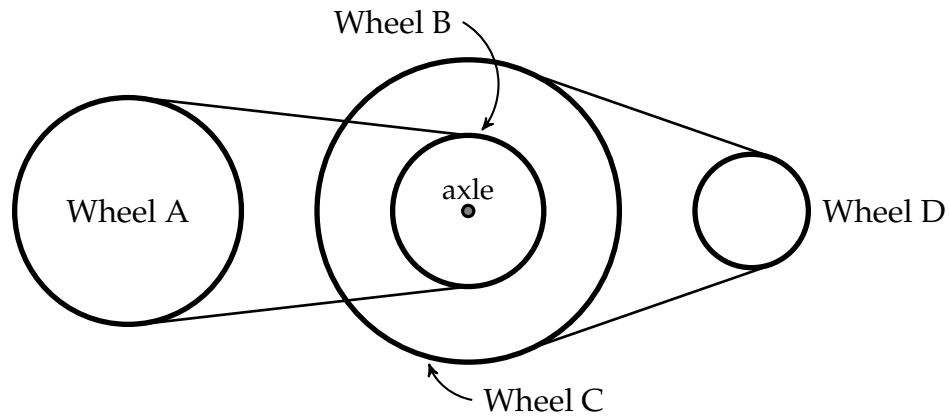
Wheel A has a radius of 7 feet and rotates at a speed of 6 revolutions per minute.

Wheel B has a radius of 4 feet, Wheel C has a radius of 8 feet, and Wheel D has a radius of 3 feet.

How many seconds does it take Wheel D to make a complete rotation?

5. Tori and Harry are both running **counter-clockwise** around a circular track of radius 10 meters. Tori begins at the northernmost point and Harry begins at the easternmost point. Harry runs faster.

   (a) **[4 points]** Tori first reaches the southernmost point after 8 seconds.

   What is Tori's speed, in meters per second?
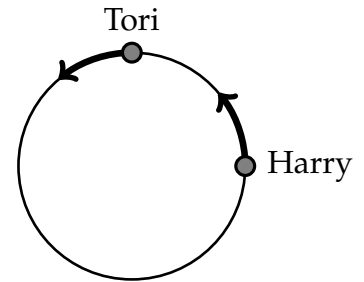
   Tori

   Harry

   (b) **[6 points]** Harry begins running at the same time as Tori, and catches up to her in 11 seconds.
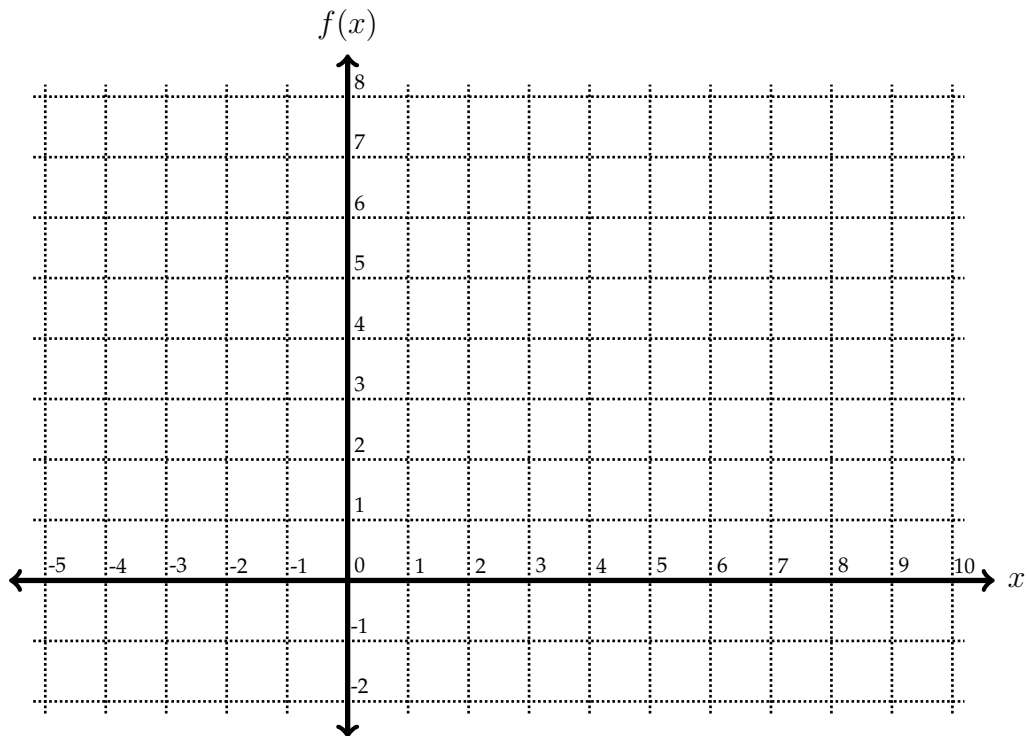
   What is Harry's speed, in meters per second?

   (c) **[5 points]** Impose a coordinate system with units in meters and the origin at the center of the circle. After 80 seconds, what are Harry's coordinates?

6. Consider the following multipart function:

$$f(x) = \begin{cases} x^2 + 6x + 8 & \text{if } -4 \le x < -1 \\ 3\sin\left(\dfrac{2\pi}{5}(x+1)\right) + 4 & \text{if } -1 \le x < 9 \end{cases}$$

(a) **[6 points]** Sketch a graph of $f(x)$. Label your graph clearly.

$f(x)$

(b) **[7 points]** Find *all solutions* to the equation $f(x) = 2$.

7. Chloë and and Joë are walking around the coördinate plane. They both begin walking at the same time, in straight lines at constant speeds.

   (a) **[3 points]** Chloë starts at $(-2, -3)$ and walks east at a speed of 4 units per second.

   Give parametric equations for Chloë's coördinates after t seconds.

   (b) **[4 points]** Joë begins at the point $(6, 3)$ and walks towards the point $(14, -5)$, reaching it in 4 seconds.

   Give parametric equations for Joë's coördinates after t seconds.

   (c) **[5 points]** When are Chloë and Joë closest together?

8. Let $f(x)$ be the linear-to-linear rational function with an x-intercept of 5 and a y-intercept of -4, passing through the point $(35, -6)$.

(a) **[7 points]** Write a formula for $f(x)$.

$$f(x) = \frac{-28x + 140}{5x - 35}$$

(b) **[2 points]** Write the domain and range of $f(x)$.

Domain: $x \neq 7$ Range: $y \neq -\dfrac{28}{5}$

(c) **[4 points]** Solve the equation $f(f(x)) = 2$.

$$x = \frac{905}{151}$$