Math 583B **Research in Geometric Analysis** **Spring 2005**

Syllabus

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Spring 2005
Monday & Wednesday 11:30–12:45

Goals of the course

Research in mathematics requires a number of skills. The most obvious one is having creative and original mathematical ideas; this skill requires a certain amount of talent and years of hard work, and probably cannot be taught. But there are several other skills that *can* be taught, such as reading and absorbing research papers, giving mathematical lectures, and writing mathematical papers. This course aims to give you some directed practice in the latter three skills, in the hope that it will stand you in good stead when you research and write your dissertation, and later when you write research papers of your own. Along the way, you will be introduced to some of the most significant contributions to geometric analysis of the past several decades.

Format of the course

This course will be run as a seminar, in which students will give lectures on recent research papers in geometric analysis. I'll distribute a list of important research papers in geometric analysis from approximately the past 50 years, organized into "clusters" of papers that are related by a common theme. You will work in *teams* of two (or perhaps three) students each. Each team will choose two clusters from the list: one early in the term, and the second after you've finished your first series of talks. (If you want to rearrange your teams after your first talks, that's OK with me.) Each team will give a sequence of one-hour lectures on each cluster.

Most 75-minute class meetings will be structured as follows: 60 minutes for a student lecture, followed by 15 minutes of class discussion during which the members of the class will evaluate and discuss the preceding lecture.

Requirements

Lectures: You'll lecture about two clusters in class (two hours per cluster). See the section on "Your lectures" below for details.

Term paper: Your team will write an expository paper based on the subject of the first of your two lecture series. See the section on "Your term paper" below for details.

Attendance and lecture evaluations: Class attendance is required. You must get permission from me (in advance, if it's not a dire emergency) if you have to miss class for some unavoidable reason. After each class in which a presentation is given, you must fill out and turn in an evaluation of that day's presentation.

Paper evaluations: You must read the first drafts of the papers of one other team in the class, and give them written evaluations on forms which will be provided. See the section on "Your term paper" below.

Choosing lecture topics

Each of you will give one lecture during the first half of the quarter, and a second lecture on a different topic during the second half. I will announce the lecture schedule for the first half of the quarter before classes start.

For each lecture, your team should choose one of the “clusters” of papers on the list. Please let me know of your choice approximately three weeks before the date of your lectures. You may choose any cluster you like, except one that has already been chosen by another team, one that contains a paper you’ve already read, or one that is integrally related to thesis research you’ve already begun. The idea is to stretch yourself a little.

Your lectures

Each “lecture series” will consist of a coherent sequence of one-hour talks, one by each team member, on the subject of a single cluster from the list of papers: a sort of “mini-course”. As far as possible, we’ll arrange things so that each team will lecture in two successive classes, Monday and Wednesday of the same week. Taken together, your lecture series should be an exposition of some significant ideas from the cluster of papers, similar to a (good) colloquium talk, aimed primarily at the other members of the class.

You’ll be responsible for searching out and reading any other papers or books that are needed to understand the background of the papers in your cluster. I’ll be happy to make suggestions if you get stuck. You should also seek out material that’s been published more recently than the papers on the list, in order to get a sense of where the field has come. The MATHSCI database will be an indispensable research tool—learn how to use it, especially the Reference Citations and Review Citations. If you are not thoroughly familiar with these tools, you can ask one of the math librarians (Martha Tucker or Mary McDonald) for a tutorial session on their use.

Most of these papers are long, contain a lot of mathematics, and require a lot of background to understand thoroughly. In fact, any one of the clusters on the list could easily serve as the subject matter for a full-year graduate course. I don’t expect that any of you, at this stage of your education, will have the time or the background to completely digest any of them. So don’t be intimidated if you don’t understand very much the first time through. In any case, two hours is nowhere near long enough to cover any of these subjects in detail. The idea is to focus on a *few* ideas from the cluster that have significance to you, and that you believe will have significance to your audience. Your focus can be, for example, a crucial lemma; an important technique; a particular application; the statement of the main theorem; or a significant special case.

Put together a series of talks that will be accessible to an audience with no background in the specific subject. It is perfectly acceptable to leave out details, if you tell us clearly what kinds of things you are leaving out. It’s also acceptable to say that you don’t fully understand certain parts of the subject, as long as they’re not central to what you’re telling us about. *But be careful to avoid being vague or dishonest!* Telling us you’re leaving out some technical hypotheses of a theorem is one thing; stating a false theorem is another.

A good way to plan your talk is to imagine that you have been invited to give a colloquium talk on this subject. Your audience will include graduate students as well as experienced researchers (me!), and you have to pick some aspect of the papers to talk about that will be both accessible and interesting to this audience. Thinking about it in this way will help you consolidate your understanding of what is important about the paper.

Each talk in the series must be less than or equal to 60 minutes long. *This is no joke: I will stop you at 12:30!* Therefore it is essential that you *practice* your talk. It's a good idea to have your teammate(s) listen to your practice talk, since they might ask questions and make suggestions that will help you polish your final performance. It's also a good idea to design each talk to last no more than 50 minutes, so that you have extra time in case people ask unexpected questions. You should always get to class on time, since the speaker will want to start promptly at 11:30.

After each talk, we will spend the remaining 15 minutes discussing the talk: what you liked or didn't like about it, how well it accomplished its goals, what parts you didn't understand, what other questions it raised in your minds, how it might be improved. Afterwards, you'll each be given a lecture evaluation form to fill out.

Your term paper

After your first series of talks, your team will write an expository paper about the same subject as the first lecture series. The paper should be written in chapters, one chapter written by each member of the team, with each chapter no fewer than 8 and no more than 16 pages long (as printed out in LaTeX/report style, or equivalent). The paper should clearly indicate who wrote each chapter.

Your paper should be aimed at the same audience as your talk. It should contain essentially the same information as your lectures, but with somewhat more detail. Look at the expository articles about mathematical research topics in the *AMS Bulletin* or the *AMS Notices* to get an idea of appropriate styles.

You'll write (at least) three drafts of your paper before you hand in the final version. Here is how they will go:

First draft: With the other member(s) of your team, plan the structure of your paper, with each person responsible for one chapter on the subject that he or she talked about in class. Then write a first draft of the paper, consulting with the other member(s) of your team as you go along to ensure consistency of notation, terminology, and organization. Once the first draft is complete, give copies of it to the members of another team for initial peer evaluation. Every team must evaluate exactly one other team's paper. Each of the members of the other team will read the first draft and fill out an evaluation form (which I will provide later in the term). They will then return the draft and their evaluations to you.

Second draft: Based on the feedback you get from the peer evaluation forms, you'll revise your paper. The second draft must be turned in to me, together with the first draft and peer evaluation forms, by Friday, May 13, at 5:00 PM. I will return it to you with my comments.

Final version: Based on the feedback you get from me, you'll revise your paper once more and create a final version. Two copies of the final version must be turned in to me, together with both previous drafts and all evaluation forms, by Wednesday, June 1, at 5:00 PM.

Your paper (all three drafts) must be typed using TeX. I don't care whether you use LaTeX or some other flavor of TeX, just as long as you use TeX in some form.

TEX is the accepted standard for producing mathematical papers and books, and most journals accept electronic submission of articles in TEX format. Thus TEX is something you should learn eventually for professional use, and now is as good a time as any.

If you have never used TEX before, I strongly recommend using LATEX together with the `amsmath` package. This combination is relatively easy to learn, creates the best-looking output, and is accepted by most journals. Basic LATEX is documented in the book *LATEX: A Document Preparation System* (1994 edition), by Leslie Lamport, which is on reserve in the Math Research Library. The `amsmath` package is a collection of LATEX macros for improving the structure and appearance of mathematical papers, created by the American Mathematical Society; the user's guide will be linked from the course web page, along with some other useful documents.

If you have some good reason why using TEX is not practical for you, come talk to me about it, and I might allow you to use an alternative typing method.

Evaluation and grading

Your work in this course will be evaluated in detail both by me and by other members of the class. This is for your own benefit. Your grade, however, will be based solely on completion of the assigned work. If you complete all the requirements described in this syllabus on time, you will receive a 4.0; if not, your grade will be reduced accordingly. In order to get a satisfactory grade in the course (3.0 or higher), you have to give both lectures and turn in the final version of your term paper.

Final remarks

This is going to be a hard course, and you will feel a lot of pressure, but I'll do my best to help you survive it and to learn a lot in the process.

Good luck!