Math 544/5/6 **Topology and Geometry of Manifolds** **2006–2007**

SYLLABUS

Lectures: MWF 1:30–2:20
Padelford C-36

Instructor: Jack Lee
Padelford C-546, 206-543-1735
lee@math.washington.edu

Teaching Assistant: Travis Kopp
Padelford C-111
koppt@math.washington.edu

Course Web site: *http://www.math.washington.edu/~lee/Courses/544-2006*
(or from the Math Department home page,
Class Web Pages → Math 544)

Textbooks: I will be writing the first draft of new editions of my *Introduction to Topological Manifolds* and *Introduction to Smooth Manifolds*, which will be made available approximately one chapter per week. The cost for the fall quarter is $30 per copy, payable to Mary Sheetz in the Math Department Office (PDL C-138).

General description:

Manifolds are arbitrary-dimensional generalizations of curves and surfaces—spaces that locally look like Euclidean space but globally may not, just as the sphere locally looks like the plane but is globally very different. They are the basic subject matter of differential geometry, but also play a role in many other branches of pure and applied mathematics. In the fall quarter we will concentrate on the *topology* of manifolds, i.e., properties that are invariant under continuous deformations. The main goals are to understand the fundamental group, covering spaces, and the classification of compact surfaces. Along the way, we will take the opportunity to study topology in somewhat greater generality, not just on manifolds.

The winter and spring quarters will be devoted to the study of *smooth manifolds*, on which derivatives of functions and maps make sense. We will study the basic flora and fauna that live on them: tangent vectors and covectors, submanifolds, vector and tensor fields, flows, Lie derivatives, Riemannian metrics, differential forms, orientations, de Rham cohomology. We will also examine the basic examples of manifolds with geometric structures, such as Lie groups, foliations, Riemannian manifolds, and symplectic manifolds.

Prerequisites:

The study of manifolds draws on a variety of areas of mathematics that are (hopefully) part of your previous education. It is not necessary to have complete command of all the topics listed here. But if any are completely missing from your background, or your skills in several have faded, you probably will find this course even more challenging than some of our other first year grad courses. In particular, you probably should not take Manifolds if you have such gaps in your background and this is your first year of graduate school. See me if you have questions about your preparation for the course.

For Fall quarter

- *Set Theory*: Operations on sets, functions, equivalence and order relations, number systems and cardinality, the axiom of choice. References: *Principles of Mathematical Analysis* by Rudin, Chapter 1; *Mathematical Analysis* by Apostol, Chapters 1 and 2; Appendix to [ITM].
- *Analysis*: Metric spaces; convergence and continuity; open and closed sets; interior, exterior, and boundary; compactness. Reference: *Principles of Mathematical Analysis* by Rudin, Chapters 2,3,4; *Mathematical Analysis* by Apostol, Chapters 3 and 4; Appendix to [ITM].
- *Algebra*: Elementary group theory, homomorphisms, isomorphisms, subgroups, normal subgroups, permutation groups, cyclic groups, cosets, quotient groups. Reference: *Abstract Algebra: An Introduction* by Hungerford, Chapter 7; *Abstract Algebra* by Herstein, Chapters 1–3; Appendix to [ITM].

For Winter and Spring quarters

- *Topology*: All the material covered in Math 544. Reference: [ITM].
- *Linear algebra*: Abstract vector spaces, subspaces, bases, dimension, matrices, determinants, change of basis formulas, linear maps, kernel and image, norms and inner products, orthonormal bases. Reference: any rigorous linear algebra text, such as *Linear Algebra* by Friedberg, Insel, and Spence; Appendix to [ISM].
- *Multivariable calculus*: Partial derivatives; the total derivative as a linear approximation; Taylor's formula in several variables; multiple integrals and the change of variables formula; gradient, divergence, and curl; the theorems of Green, Gauss, and Stokes; uniform convergence. References: *Basic Multivariable Calculus* by Marsden, Tromba, and Weinstein; *Principles of Mathematical Analysis* by Rudin, Chapters 5,6,7; *Mathematical Analysis* by Apostol, Chapters 12–14; Appendix to [ISM].
- *Differential equations*: Basic facts about existence and uniqueness of solutions to ODEs; elementary techniques for solving first-order equations and systems at the level of Math 307 and 309. Reference: *Elementary Differential Equations and Boundary Value Problems* by Boyce and DiPrima.

Requirements:

A homework assignment will be given out once a week, due one week later. A typical homework assignment will consist of the following:

1. *Reading:* Typically, you will be given approximately one chapter to read each week. I will expect you to read through the chapter quickly before the relevant lectures, and then to reread it carefully after the lecture.

2. *Reading Report:* Each week, you're required to submit a short reading report to the EPost discussion group for this course. (Follow the link on the class web page.) Your report must include at least two paragraphs:

   - Briefly describe the most important idea(s) in this week's reading assignment, in your judgment.
   - List one or two questions that the reading raised in your mind.

   Your questions might address such issues as why something is defined the way it is, how a given concept might be of use, something you'd like to learn more about, or something that made you feel "stuck." You may respond (respectfully!) to other students' postings if you wish. In these reports, *there is no such thing as a stupid qustion!*

   Your report may also include any other comments or questions you'd like to raise concerning the course, including the lectures, classwork, reading, homework, or exams. If you wish to write about specific homework problems, please confine your comments to general questions and suggestions about how to get started.

   The due date for each reading report will be announced with the homework assignment. Part of your grade will be based on the reading reports. (The only thing that will be graded is whether you've submitted them; as long as you make a good-faith effort to include the two items mentioned above, the content of your reports won't affect your grade.) You may skip at most two weeks to get full credit.

3. *Exercises:* Scattered throughout the textbooks are various *exercises* (as distinct from the *problems* at the ends of the chapters). I expect you to do (or at least figure out how to do) *all of them* as you read. Most of these are relatively routine, and will not be collected or graded, but nevertheless understanding them is important to the continuity of the text and the lectures.

4. *Written exercises and problems:* Each week a few exercises and/or problems will be assigned for you to write up and hand in. These problems are the heart of the course.

5. *Optional problems:* Sometimes I will list a few problems as optional. Some of these will carry the ideas from the course further, or lead you through proofs that we don't have time to cover. Some of them are just particularly difficult examples. If you have time, and if you really want to understand the subject, I encourage you to try as many of these as you can. I'll be more than happy to discuss them with you outside of class. (They won't officially count in your numerical grade, but if your grade is on a borderline they can influence which way it goes.)

Homework guidelines:

I encourage you to work with other students. Discussing problems and ideas with your classmates is one of the best ways to learn the material. But when writing up solutions to hand in, *you must write your own solutions in your own words.* If you collaborate on any assignment, you must list the names of any people with whom you collaborated on that assignment. Also, please do not look at anyone's complete written solution (including published proofs) before turning in your homework. *We should not see evidence in the homework that you are going beyond discussing the problems to studying someone else's written solutions.*

You may freely cite results of Exercises from earlier in the book. (For this purpose, the Appendix is considered to be earlier than all the other chapters.) Unless otherwise stated, you may only use another Problem if it has been previously assigned, or if you give its solution. If you look up and use something proved in another book, cite it.

Here are my expectations regarding the mechanics of writing up homework assignments:

- **Stapled:** Please staple the pages of each assignment together.
- **Identification:** Make sure the first page of each homework packet is clearly labeled with your name and the assignment number.
- **In order:** Arrange your solutions in numerical order, just as they appear on the assignment sheet, with each problem starting on a new page. Problems that are out of order might not get credit.
- **Legible:** Write your answers neatly and legibly, not too small, with as few erasures or crossouts as possible. Be sure to distinguish clearly between similar symbols, such as $l/1$, $b/6$, $\in/\epsilon$, $g/q/9$, h/n, p/ρ, r/γ, $s/5$, $t/+$, v/ν, $x/\times$, $y/4$, $z/2$, $\subset/C$, $\cup/U$, and uppercase/lowercase letters. Unless mathematical ideas spring fully and impeccably realized from your pen, your first draft is not acceptable.
- **White space:** Don't be stingy with white space. Leave one-inch margins on all sides of your pages.

I welcome computer-typeset submissions from those who are comfortable producing mathematical homework assignments by computer. If you do use a computer, I recommend LaTeX. I'm also happy to accept handwritten assignments, as long as they are neat and legible.

Exams:

At the end of each quarter there will be a take-home final exam. It will be somewhere between a long homework assignment and a manifolds prelim, covering material from the entire quarter.

Grading:

Your grade will be based 2/3 on the required homework problems (with the reading reports counting as one homework assignment) and 1/3 on the final exam. I will not decide on the exact scale until the end of the quarter, but when I've taught this course in past years an average of about 90% yielded a 4.0 and 70% a 3.0, with scores in between linearly interpolated. The minimum grade that can be counted toward a degree in mathematics is 3.0.