

# CORRECTIONS TO Introduction to Complex Manifolds

BY JOHN M. LEE  
NOVEMBER 22, 2025

(10/27/25) **Page 6, last displayed equation:** Replace this equation by

$$\varphi_\alpha \circ \varphi_\beta^{-1}(z^1, \dots, z^n) = \left( \frac{z^1}{z^{\alpha+1}}, \dots, \frac{\widehat{z^{\alpha+1}}}{z^{\alpha+1}}, \dots, \frac{1}{z^{\alpha+1}}, \dots, \frac{z^n}{z^{\alpha+1}} \right).$$

Then in the line below that equation, change  $\alpha$  to  $\alpha + 1$  and  $1/z^\alpha$  to  $1/z^{\alpha+1}$ .

(11/19/24) **Page 15, last displayed equation:** In the formula for  $a_{k_1 \dots k_n}$ , insert a factor of  $1/(2\pi i)^n$  on the right-hand side.

(10/10/25) **Page 21, proof of Proposition 1.35:** Several changes need to be made to this proof. First, replace the sentence just above the third display by the following: “Applying this in turn to each contour integral in the above formula and using the fact that  $|\zeta^n - z^n| \geq |\zeta^n - p^n| - |z^n - p^n|$  gives the following estimate for  $z \in \overline{D}_{r/2}^n(p)$ :

$$\left| \frac{\partial f_k(z)}{\partial x^j} \right| \leq \frac{2^{n+1}C}{r}.”$$

Next, two lines below that display, change  $D_r^n(p)$  to  $\overline{D}_{r/2}^n(p)$ . In that same paragraph, delete the sentence beginning with “By continuity,” and in the next-to-last line, change  $\overline{D}_r^n(p)$  to  $\overline{D}_r^n(p)$ . Finally in the last paragraph of the proof, change “some polydisk  $D_r^n(p)$  such that  $\overline{D}_r^n(p) \subseteq U$ ” to “some polydisk  $\overline{D}_{r/2}^n(p)$  such that  $\overline{D}_r^n(p) \subseteq U$ .”

(6/15/24) **Page 25, fourth full paragraph, lines 4 and 8:** Change  $\sigma \circ \pi$  to  $\pi \circ \sigma$  (twice).

(6/15/24) **Page 25, fourth full paragraph, line 7:** Delete the repeated phrase “*rough (local or global) section of E* is a map  $\sigma : U \rightarrow E$ .”

(6/15/24) **Page 26, line 8 from the bottom:** Change  $[Y_2, X_1]$  to  $[Y_1, X_2]$ .

(11/2/25) **Page 27, equation (1.7):** In the first line, change the second occurrence of  $\partial f / \partial x^j$  to  $\partial f / \partial y^j$ .

(6/19/24) **Page 46, line 2:** Change  $(\partial \overline{F}^k \partial \overline{z}^k)$  to  $(\partial \overline{F}^k / \partial \overline{z}^j)$ . [Insert missing slash and fix incorrect index.]

(11/27/24) **Page 50, Example 2.21, second line:** Change  $\subseteq$  to  $\in$ .

(3/18/25) **Page 57, proof of Lemma 2.35, first paragraph:** Change  $\sum_{j,k}$  to  $\sum_{i,j}$  (twice).

(2/17/25) **Page 59, line 3:** Change “projective hypersurface” to “projective variety.”

(2/17/25) **Page 59, Example 2.38, second line:** Change “holomorphic function” to “holomorphic polynomial.”

(6/17/24) **Page 60, line 2 from the bottom:** Change  $[x_0, \lambda_j(x_0), 1]$  to  $[1, x_0, \lambda_j(x_0)]$ .

(3/19/25) **Page 62, proof of Theorem 2.43, first paragraph, next-to-last line:** After  $\mathbb{C}^{n+1} \setminus \{0\}$ , insert “(see Lemma 3.38 below).”

(11/2/25) **Page 63, first full paragraph, third sentence:** Change the last part of that sentence to “... the algebraic variety  $V_r$  defined by the homogeneous polynomial  $r(p) = R(\partial_0 p, \dots, \partial_n p)$ , noting that the coefficients of  $\partial_0 p, \dots, \partial_n p$  are linear functions of the coefficients of  $p$ .” Then in the rest of that paragraph, change  $V_R$  to  $V_r$  (three times), and in the displayed equation change  $R(w^0 p + w^1 q)$  to  $r(w^0 p + w^1 q)$ .

(3/18/25) **Page 66, second paragraph, line 3:** Change “points  $p_j$  in  $K$ ” to “points  $p_j$  in  $\widehat{K}$ .”

(1/15/25) **Page 77, proof of Lemma 3.14, second paragraph:** Change “such a subsequence” to “such a sequence.”

(7/11/24) **Page 78, proof of Theorem 3.13, last line of the first paragraph:** Change  $\{\sigma_k\}_{k=1}^\infty$  to  $\{\sigma_i\}_{i=1}^\infty$ , and  $\|\sigma_k\|$  to  $\|\sigma_i\|$ .

(7/11/24) **Page 79, first line:** Change  $\mathbb{S}^{2n-1}$  to  $\mathbb{S}^{2k-1}$ .

(7/11/24) **Page 79, second paragraph:** Change  $\sigma_k$  to  $\sigma_i$  (four times).

(6/18/24) **Page 80, Example 3.21, lines 3 & 4:** “Holomorphic” is misspelled.

(2/3/25) **Page 81, first displayed equation:** Change the matrix to the following:

$$\tau_j^i = \begin{pmatrix} \alpha_j^i & \beta_j^i \\ 0 & \gamma_j^i \end{pmatrix}.$$

(6/18/24) **Page 91, two lines above the displayed equation:** Delete repeated “on.”

(6/18/24) **Page 94, second-to-last line:** Change “Theorem 3.41” to “Theorem 3.39.”

(5/26/25) **Page 95, last paragraph, line 5:** Change  $\mathbb{C}\mathbb{P}^n$  to  $\mathbb{C}\mathbb{P}^m$ .

(7/10/24) **Page 97, last line of the proof:** Change “with  $\sigma = s_j$ ” to “with  $\sigma = s_j - f_j(p)s_0$  (so that  $\hat{\sigma} = f_j - f_j(p)$ ).”

(6/18/24) **Page 110, second full paragraph, first line:** Change  $z_1$  and  $b_1$  to  $z^1$  and  $b^1$ , respectively, so the formula reads  $\bar{D}_R(z^1) \supseteq D_\varepsilon(b^1)$ .

(3/19/25) **Page 111, last displayed equation:** Swap  $f \leftrightarrow g$ ,  $f_1 \leftrightarrow g_1$ , and  $f_2 \leftrightarrow g_2$ .

(7/16/24) **Page 112, line 4:** Change  $\{0, \dots, q\}$  to  $\{0, \dots, n\}$ .

(1/13/25) **Page 112, second full paragraph, first line:** Change  $\mathcal{E}_k^{0,q}(M)$  to  $\mathcal{E}_k^{0,q}(U)$ .

(7/16/24) **Page 113, last line of the proof:** Change  $\bar{\partial}(\sum_I' \alpha_I \wedge \sigma_I)$  to  $\bar{\partial}((-1)^p \sum_I' \alpha_I \wedge \sigma_I)$ .

(11/19/25) **Pages 113–114, proof of Corollary 4.15:** This proof is incorrect. Replace the entire proof by the following:

**Proof.** Let  $k = p + q$  and let  $x \in M$ . Because  $\theta$  is a closed  $k$ -form, in a neighborhood of  $x$  there is a complex  $(k - 1)$ -form  $\eta$  such that  $d\eta = \theta$  by the ordinary Poincaré lemma. (The Poincaré lemma applies to complex-valued forms by applying it separately to the real and imaginary parts.) Write  $\eta = \sum_{r+s=k-1} \eta^{(r,s)}$ , with  $\eta^{(r,s)}$  of type  $(r, s)$ .

We will show by induction on  $r$  that for  $0 \leq r \leq p - 1$ , there exist forms  $\gamma^{(r-1,s)}$  and  $\gamma^{(r,s-1)}$  on a (possibly smaller) neighborhood of  $x$  such that  $\eta^{(r,s)} = \partial\gamma^{(r-1,s)} + \bar{\partial}\gamma^{(r,s-1)}$ . For  $r = 0$ , the  $(0, k)$ -part of  $d\eta = \theta$  shows that  $\bar{\partial}\eta^{(0,k-1)} = 0$ , so by the  $\bar{\partial}$ -Poincaré lemma there is a  $(0, k - 2)$ -form  $\gamma^{(0,k-2)}$  on a neighborhood of  $x$  such that  $\bar{\partial}\gamma^{(0,k-2)} = \eta^{(0,k-1)}$ . With  $\gamma^{(-1,k-1)} = 0$ , this proves the claim for  $r = 0$ .

Supposing the claim has been proved for some  $r < p - 1$ , the  $(r + 1, s)$ -part of  $d\eta = \theta$  (together with the fact that  $\partial^2 = 0$ ) shows that

$$0 = \partial\eta^{(r,s)} + \bar{\partial}\eta^{(r+1,s-1)} = \bar{\partial}(\eta^{(r+1,s-1)} - \partial\gamma^{(r,s-1)}).$$

By the  $\bar{\partial}$ -Poincaré lemma again, there exists  $\gamma^{(r+1,s-2)}$  such that  $\eta^{(r+1,s-1)} - \partial\gamma^{(r,s-1)} = \bar{\partial}\gamma^{(r+1,s-2)}$ , which completes the induction step. The same argument applied to  $\bar{\eta}$  shows that for  $p \leq r \leq k - 1$ , there exist forms  $\rho^{(r-1,s)}$  and  $\rho^{(r,s-1)}$  on a neighborhood of  $x$  such that  $\eta^{(r,s)} = \partial\rho^{(r-1,s)} + \bar{\partial}\rho^{(r,s-1)}$ .

Now the  $(p, q)$ -part of  $d\eta = \theta$  gives

$$\theta = \partial\eta^{(p-1,q)} + \bar{\partial}\eta^{(p,q-1)} = \partial\bar{\partial}\gamma^{(p-1,q-1)} + \bar{\partial}\partial\rho^{(p-1,q-1)},$$

so taking  $\alpha = -i\gamma^{(p-1,q-1)} + i\rho^{(p-1,q-1)}$  proves the result. If  $\theta$  is real, we can choose  $\eta$  to be real, which means  $\eta^{(p,p-1)} = \eta^{(p-1,p)}$ ; and then we can choose  $\rho^{(p-1,q-1)} = \overline{\gamma^{(p-1,q-1)}}$ , so  $\alpha$  is real.  $\square$

(6/18/24) **Page 115, third line:** Change “write  $\beta$ ” to “write an arbitrary vector-valued form  $\beta$ .”

(7/19/24) **Page 115, five lines below equation (4.17):** Change  $E^{q+q'}(M; E)$  to  $E^{q+q''}(M; E)$ .

(6/18/24) **Page 115, eight lines below equation (4.17):** Change  $\omega \in \mathcal{E}^q(\text{End}(E))$  to  $\omega \in \mathcal{E}^q(M; \text{End}(E))$ .

(7/19/24) **Page 116, third line from the bottom:** Change  $\tilde{\sigma}^k \tilde{s}_k$  to  $\tilde{\sigma}^k \otimes \tilde{s}_k$ .

(6/18/24) **Page 123, three lines above Example 5.2:** Change  $S_k(U)$  to  $\mathcal{S}_k(U)$ .

(3/16/25) **Page 131, proof of Lemma 5.8:** In the fourth line, replace  $U$  by  $B$ . In lines 5 and 6, replace  $V \cap B$  by  $V$ .

(3/16/25) **Page 145, Problem 5-2:** Change “abelian groups” to “abelian groups, rings, vector spaces, or modules over a sheaf  $\mathcal{R}$ .”

(6/17/24) **Page 145, Problem 5-7:** Change  $\mathcal{H}om$  to  $\mathcal{H}om$  (three times). [This printing error, which also occurs on page 191, should be fixed in later printings and later versions of the ebook.]

(6/18/24) **Page 145, Problem 5-8(a):** Change “Exercise 5.6” to “Problem 5-1.”

(4/7/25) **Page 148, proof of Lemma 6.1:** In the displayed equation, change all instances of  $p + 1$  to  $p + 2$  (five times). Here’s the correct formula:

$$\begin{aligned} (\delta\delta c)_{\alpha_0 \dots \alpha_{p+2}} &= \sum_j (-1)^j (\delta c)_{\alpha_0 \dots \hat{\alpha}_j \dots \alpha_{p+2}} \Big|_{U_0 \cap \dots \cap U_{p+2}} \\ &= \sum_{j,k} (-1)^{j+k} c_{\alpha_0 \dots \hat{\alpha}_k \dots \hat{\alpha}_j \dots \alpha_{p+2}} \Big|_{U_0 \cap \dots \cap U_{p+2}} \\ &\quad + \sum_{j,k} (-1)^{j+k-1} c_{\alpha_0 \dots \hat{\alpha}_j \dots \hat{\alpha}_k \dots \alpha_{p+2}} \Big|_{U_0 \cap \dots \cap U_{p+2}}, \end{aligned}$$

(4/7/25) **Page 156, three lines above equation (6.11):** Change  $\rho^\#$  to  $\rho^*$ .

(7/15/24) **Page 156, two lines above equation (6.11):** Change “ $\mathcal{A}$  and  $\mathcal{B}$ ” to “ $\mathcal{B}$  and  $\mathcal{C}$ .”

(12/11/24) **Page 157, line 5:** Change  $C^{p-1}(\mathcal{W}; \mathcal{E})$  to  $C^{p-1}(\mathcal{W}; \mathcal{B})$ .

(12/11/24) **Page 157, third paragraph, first line:** Replace  $\beta_* \circ \mathcal{F}$  by  $\mathcal{F} \circ \beta_*$ .

(4/7/25) **Page 158, first displayed equation:** Add a restriction to  $V_{x_0} \cap \dots \cap V_{x_p}$  on the right-hand side:

$$\tilde{b}_{x_0 \dots x_p} = b_{\rho(x_0) \dots \rho(x_p)} \Big|_{V_{x_0} \cap \dots \cap V_{x_p}}.$$

(4/7/25) **Page 158, line below the first displayed equation:** Change  $b$  to  $\tilde{b}$  (twice).

(4/7/25) **Page 162, Example 6.12, second paragraph:** Insert “and every integer  $p \geq 1$ ” at the end of the first sentence of the paragraph.

(3/19/25) **Page 163, Example 6.13(c):** In the first two lines, change “Every skyscraper sheaf” to “Every skyscraper sheaf on a Hausdorff space,” and “on a topological space” to “on a Hausdorff space.”

(3/19/25) **Page 165, four lines above Proposition 6.15:** Change “cohomology class” to “homology class.”

- (4/7/25) **Page 169, line 2:** Change “0-chain” to “0-cochain.”
- (3/19/25) **Page 169, third line from the bottom:** Add missing left parenthesis in  $H_{\text{Sing}^+}^k(M; G)$ .
- (4/7/25) **Page 170, last paragraph:** Eliminate property (iv) (it is redundant), and delete the rest of the last sentence on the page beginning with “and (iv) because ...”
- (4/7/25) **Page 171, line above equation (6.24):** Change (iv) to (i).
- (11/27/24) **Page 178, statement of Corollary 6.26, last line:** Change “cohomologous” to “homologous.”
- (7/15/24) **Page 179, third line from the bottom:** Change “cycles” to “cocycles.”
- (7/15/24) **Page 181, line 10:** Change  $\delta^*([L])$  to  $\delta_*([L])$ .
- (3/19/25) **Page 183, line 3:** Change  $H^2(M; \mathcal{O}^*)$  to  $H^1(M; \mathcal{O}^*)$ , and delete the spurious colon after the arrow.
- (6/17/24) **Page 191, Problem 6-9(b):** Change  $\mathcal{H}m$  to  $\mathcal{H}om$ .
- (6/17/24) **Page 191, Problem 6-10, first paragraph:** Change  $\text{St } v_0 \cap \cdots \cap \text{St } v_p$  to  $\text{St } v_0 \cup \cdots \cup \text{St } v_p$ .
- (6/17/24) **Page 193, second paragraph, last line:** Change “Chapter 3” to “Chapter 6.”
- (7/19/24) **Page 198, first full paragraph, second line:** Change  $[0, b]$  to  $[t_0, b]$ .
- (6/17/24) **Page 198, Proposition 7.8(b):** Change  $\nabla E$  to  $\nabla \sigma$ .
- (7/19/24) **Page 199, line below (7.9):** Change “If we interpret  $\theta_j^l$  and  $\Theta_j^l$  as the local expressions for  $\text{End}(E)$ -valued forms” to “If we interpret the matrix  $\theta = (\theta_j^l)$  as the local expression for an  $\text{End}(E)$ -valued form.”
- (7/19/24) **Page 199, nine lines from the bottom:** Change  $\tilde{\theta} = \tau^{-1} d\tau$  to  $\tilde{\theta} = \tau^{-1} \theta \tau$ .
- (7/19/24) **Page 200, line below the second displayed equation:** Change  $\partial_{j+1} \sigma$  to  $\nabla_{\partial_{j+1}} \sigma$ .
- (7/19/24) **Page 201, part (iii) of the statement of Proposition 7.11:** Change “ $D^2 \alpha = \Theta \wedge \alpha$ ” to “For  $\gamma \in \mathcal{E}^q(M; E)$ ,  $D^2 \gamma = \Theta \wedge \gamma$ .”
- (7/20/24) **Page 203, lines 2 & 3:** Change “a section of  $\mathcal{E}^1(\text{End}(E))$ ” to “an element of  $\mathcal{E}^1(M; \text{End}(E))$ .”
- (7/19/24) **Page 203, second line after the proof of Proposition 7.13:** Change “fiber metric on  $\nabla$ ” to “fiber metric on  $E$ .”
- (6/17/24) **Page 204, paragraph above Theorem 7.14:** Change “Chapter 5” to “Chapter 6.”
- (6/17/24) **Page 206, second line of the Remark:** Change  $\delta^*([L])$  to  $\delta_*([L])$ .
- (11/27/24) **Page 211, proof of Theorem 7.22, third line of the proof and third line from the bottom of the page:** Change  $L$  to  $L_D$  (twice).
- (11/27/24) **Page 212, equation (7.21) and the third line below it:** Change  $L$  to  $L_D$  (twice).
- (6/17/24) **Page 215, last displayed equation:** Change  $\int_0^\infty$  to  $\int_1^\infty$ .
- (6/17/24) **Page 219, Problem 7-5, second line:** Change “Problem 7-5” to “Problem 7-4.”
- (11/27/24) **Page 224, paragraph before Lemma 8.5:** Change “Hermitian structure” to “Hermitian metric.”
- (11/27/24) **Page 241, last two displayed equations:** The fractions  $\frac{1}{8}$ ,  $\frac{1}{8}$ , and  $\frac{1}{2}$  should be replaced with  $\frac{1}{16}$ ,  $\frac{1}{16}$ , and  $\frac{1}{4}$  (twice).

(11/27/24) **Page 253, Problem 8-14(a):** Replace the displayed equation by

$$B(Z, W) = \sec(\operatorname{Re} Z, \operatorname{Re} W) + \sec(\operatorname{Re} Z, \operatorname{Im} W).$$

(11/27/24) **Page 268, sentence after the first displayed equation:** Replace “is just a smooth bundle homomorphism  $P : \Gamma(E) \rightarrow \Gamma(F)$ ” by “can be identified with a smooth bundle homomorphism from  $E$  to  $F$ .”

(1/28/25) **Page 268, last displayed equation:** Change  $C\xi^2$  to  $C\eta^2$ .

(7/7/25) **Page 272, remark after Theorem 9.21:** Replace the remark by the following:

**Remark:** We have stated the Hodge theorem for oriented manifolds for convenience, because that is the only case we need, and it is the only case for which we have given an explicit computation of  $d^*$ . But this proof of the Hodge theorem actually works equally well in the nonorientable case, as you can check.

(11/27/24) **Page 273, statement of Corollary 9.22, second line:** “manifold” is misspelled.

(1/28/25) **Page 273, lines 4 & 3 from the bottom:** Change  $H^{N-k}(M; E)$  and  $H^{N-k}(M; E^*)$  to  $H^{N-k}(M; \mathbb{C})$  and  $H^{N-k}(M; \mathbb{C})^*$ , respectively.

(1/28/25) **Page 274, Lemma 9.23(a):** Change  $\mathcal{E}^{p+q}(M)$  to  $\mathcal{E}^{p,q}(M)$ .

(1/28/25) **Page 279, proof of Theorem 9.37, first displayed equation:** The last three occurrences of  $\eta$  in that equation should be changed to  $\zeta$ :

$$\int_M \eta \wedge \zeta = \int_M \bar{\partial}_E \gamma \wedge \zeta = \int_M \bar{\partial}(\gamma \wedge \zeta) = \int_M d(\gamma \wedge \zeta) = 0.$$

(11/27/24) **Page 283, last paragraph of the proof of Proposition 9.38:** Change Theorem 8.10(e) to Theorem 8.10(d).

(11/27/24) **Page 289, Corollary 9.47:** Change  $b_k$  to  $b^k$  and  $b_{k+2}$  to  $b^{k+2}$  (twice).

(11/19/25) **Pages 292–293, proof of Corollary 9.57:** Replace the proof by the following:

**Proof.** Because  $\theta$  is exact, there is a complex-valued  $(p + q - 1)$ -form  $\eta$  such that  $\theta = d\eta$ . The rest of the proof proceeds exactly like the proof of Corollary 4.15, except that in place of the  $\bar{\partial}$ -Poincaré lemma we use the Hodge–Dolbeault theorem to show that for  $0 \leq r \leq p - 1$  we have  $\eta^{(r,s)} = \partial\gamma^{(r-1,s)} + \bar{\partial}\gamma^{(r,s-1)} + \sigma^{(r,s)}$  with  $\sigma^{(r,s)}$  harmonic; and similarly  $\eta^{(r,s)} = \partial\rho^{(r-1,s)} + \bar{\partial}\rho^{(r,s-1)} + \sigma^{(r,s)}$  when  $p \leq r \leq p + q$ . Because harmonic forms are  $\partial$ -closed and  $\bar{\partial}$ -closed, the result follows.  $\square$

(11/27/24) **Page 296, proof of Theorem 9.61, line 6:** Change  $\chi(L \otimes L_{\{p\}}^k)$  to  $\chi(\mathcal{O}(L \otimes L_{\{p\}}^k))$  and  $\chi(L)$  to  $\chi(\mathcal{O}(L))$ .

(9/21/24) **Page 311, Problem 9-13:** There are several errors in the problem statement. First, replace the second sentence by the following: “Define the *dual torus* to be the group  $T^* = \operatorname{Hom}(\Lambda, \mathbb{C}^*)$  (the set of homomorphisms from the lattice  $\Lambda$  to the group of nonzero complex numbers, which is a group under pointwise multiplication).” Then in part (a), replace  $e^{2\pi i \operatorname{Im} f(v)}$  by  $e^{2\pi i f(v)}$ , and replace the displayed equation by

$$\Lambda^* = \{f \in \bar{V}^* : f(v) \in \mathbb{Z} \text{ for all } v \in \Lambda\}.$$

(9/21/24) **Page 311, Problem 9-14:** In the third line and in the displayed equation, change  $\Lambda$  to  $\Gamma$  (to be consistent with the notation in the proofs of Theorems 9.68 & 9.69).

(11/27/24) **Page 318, first line of the proof:** Change “local frame” to “local orthonormal frame.”

(11/27/24) **Page 319, statement of Lemma 10.3:** Change  $\mathcal{E}^{1,1}(M; E)$  to  $\mathcal{E}^{1,1}(M; \operatorname{End}(E))$ .

(1/28/25) **Page 321, proof of Proposition 10.5, last sentence:** Change  $F$  to  $f$ .

(1/28/25) **Page 331, expression (10.14):** Remove spurious last right parenthesis.

(1/28/25) **Page 331, last paragraph, first line:** Change  $\pi_*$  to  $\pi^*$ .

(11/27/24) **Page 333, third display:** Remove spurious last right parenthesis.

(1/28/25) **Page 335, just above the last displayed equation:** Change  $\text{Sing}_\infty^2(\tilde{M})$  to  $\text{Sing}_2^\infty(\tilde{M})$ .

(1/28/25) **Page 338, Problem 10-5, last line:** Remove spurious “that.”