

ERRATUM: *Representation-theoretic support spaces for finite group schemes*, American Journal of Math **127** (2005), 379-420.

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As pointed out to us by Rolf Farnsteiner, the results presented in our paper require a modified definition of “abelian p -point.” With this modified definition (functionally equivalent to one which we implicitly use), all of the results of our paper become valid. We make explicit this modified definition as well as those arguments which require this new definition and the modification of one proof (of Theorem 4.8) which is required.

We recall that we use the notation kG for the group algebra of a finite group scheme G , the dual of the coordinate algebra $k[G]$ of G . As defined below, a “ p -point” $\alpha : k\mathbb{Z}/p\mathbb{Z} \rightarrow kG$ is an abelian p -point (as in Definition 3.2 of [1]) with the additional condition that α factors through some subgroup scheme which is not only abelian but also *unipotent*. Among the numerous equivalent conditions on an abelian finite group scheme C to be “unipotent” (cf. [2, 8.3]), we implicitly utilize the condition that the group algebra kC be a local algebra (i.e., that C be co-connected).

Definition of p -point Let G be a finite group scheme. A p -point of G is a (left) flat map of algebras $\alpha : k\mathbb{Z}/p\mathbb{Z} \rightarrow kG$ which admits a factorization as a flat map of algebras $\alpha' : k\mathbb{Z}/p\mathbb{Z} \rightarrow kC$ followed by the map $kC \rightarrow kG$ induced by the embedding of some *unipotent* abelian group scheme $C \subset G$.

We employ precisely the same equivalence relation on p -points as in Definition 2.5 of [1], and the results of our paper are then valid provided that one takes $P(G)$ to be the space of equivalence classes of p -points of a finite group scheme G . In particular, Proposition 4.2 of [1] which asserts that every “abelian p -point” is equivalent to one factoring through a “quasi-elementary” abelian subgroup scheme (i.e., of the form $\mathbb{G}_{a(s)} \times E$ with E an elementary abelian p -group) becomes valid once we replace “abelian p -point” by p -point, thereby imposing the condition of unipotence.

As Farnsteiner has observed, the second assertion of Proposition 2.4 of [1] is not valid without the assumption that the map $\alpha : k\mathbb{Z}/p\mathbb{Z} \rightarrow kG$ factors through a unipotent abelian group scheme C . If one makes such an assumption, then the second assertion is an almost immediate consequence of the known structure of $kC \simeq k[C^*]$ (cf. [2, 14.4]). Farnsteiner further observes that the condition of unipotence is necessary for Lemma 2.10 of [1]; with our new definition of p -point which assumes unipotence, this lemma becomes redundant now that we have replaced “abelian p -point” by p -point.

Finally, Farnsteiner points out that in the proof of Theorem 4.8 the argument involving (4.8.2) establishing the inclusion $\Psi_G(P(G)_M) \subset \text{Proj}|G|_M$ requires the unipotence condition on α . Following Farnsteiner’s suggestion, we verify this inclusion as follows. Assume first that G is a quasi-elementary abelian finite group scheme of the form $\mathcal{E} = \mathbb{G}_{a(s)} \times E$. In this case, a change of coproduct of the algebra $k\mathcal{E}$ does not affect the kernel I_M of the ring homomorphism $H^\bullet(\mathcal{E}, k) \rightarrow \text{Ext}_{k\mathcal{E}}^*(M, M)$ because this equals the annihilator ideal of $\text{Ext}_{k\mathcal{E}}^*(k, M)$ as a $H^\bullet(\mathcal{E}, k)$ -module. Moreover, since \mathcal{E} is a unipotent abelian finite group scheme,

a change of coproduct does not affect $P(\mathcal{E})$ by definition and thus does not affect $\Psi_{\mathcal{E}}$. Consequently, we may replace \mathcal{E} by $\mathbb{G}_{a(r+s)}$ which has group algebra $k\mathbb{G}_{a(r+s)}$ isomorphic to $k\mathcal{E}$ (where $r = \text{rk}(E)$). Thus, the equality $\Psi_{\mathbb{G}_{a(r+s)}}(P(\mathbb{G}_{a(r+s)})_M) = \text{Proj}|\mathbb{G}_{a(r+s)}|_M$ given by Proposition 3.8 of [1] implies $\Psi_{\mathcal{E}}(P(\mathcal{E})_M) = \text{Proj}|\mathcal{E}|_M$.

For an arbitrary finite group scheme G , Proposition 4.2 of [1] tells us that any p -point of $P(G)$ lies in the image of $P(\mathcal{E}) \rightarrow P(G)$ for some quasi-elementary abelian subgroup scheme $\mathcal{E} \subset G$. Thus, the required inclusion $\Psi_G(P(G)_M) \subset \text{Proj}|G|_M$ follows from the naturality of Ψ with respect to $\mathcal{E} \subset G$ and the fact that $\text{Proj}|\mathcal{E}| \rightarrow \text{Proj}|G|$ restricts to $\text{Proj}|\mathcal{E}|_M \rightarrow \text{Proj}|G|_M$ by Theorem 1.5 of [1].

In conclusion, we express our gratitude to Rolf Farnsteiner for his identification of the necessity of assuming unipotence in our definition of p -point of a finite group scheme.

REFERENCES

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