

4/5/2019.

Defn: A Turing machine is a 7-tuple.
 $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$

eg. $NONPRIME = \{ \text{integers } n \geq 0 \text{ in binary, which is not prime} \}$
 $= \{ 1, 4, 6, 8, 9, 10, 12, \dots \}$

(1) define $i = 2$. (when $n \neq i$).

(2) Divide the input n by i . if there is no remainder. \rightarrow Accept.

otherwise, set $i = i + 1$ and repeat (2).

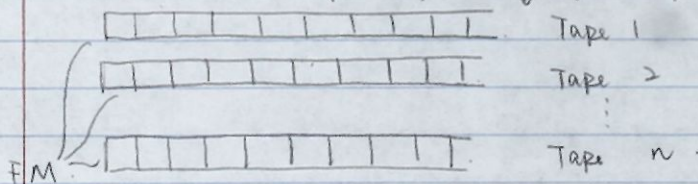
(3). if $n = i$,
reject. \rightarrow TM does recognize nonprime.

Defn: A Turing machine **decides** A if for all $w \in \Sigma^*$.

$\begin{cases} M \text{ accepts } w \iff w \in A \\ M \text{ rejects } w \iff w \notin A \end{cases}$

Multiple TM.

A TM is multiple tape. \swarrow input tape.



THM: A TM with multiple tapes is equivalent to TM with 1 tape.



Nondeterminism.

A **non-deterministic TM** is similar to a TM but at every step, the machine can proceed in multiple way simultaneously.

"it can perform multiple branches at the same time"

A power set of S is $P(S) = \text{Set of subsets of } S$.

$$P(\{0, 1, 2\}) = \left\{ \begin{array}{l} \{\emptyset\}, \{0\}, \{1\}, \{2\}, \\ \{0, 1\}, \{0, 2\}, \{1, 2\}, \\ \{0, 1, 2\} \end{array} \right\}$$

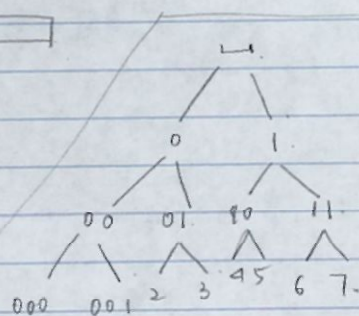
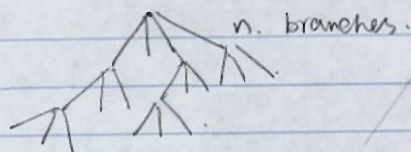
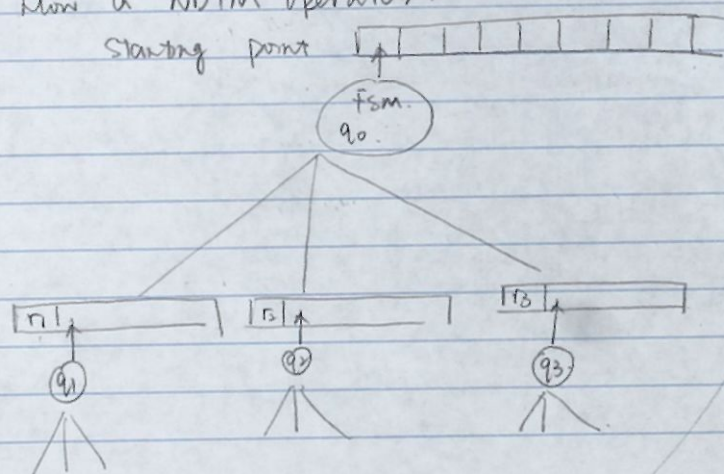
A non-deterministic TM (NDTM).

$$\delta: Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L, R\})$$

$$(q, r) \rightarrow \left\{ \begin{array}{l} (q_1, r_1, L, R), \\ (q_2, r_2, L, R), \\ \vdots \\ (q_n, r_n, L, R) \end{array} \right\}$$

eg. How a NDTM operates.

Starting point



eg. NOPRIME

(1) Let $i = 2$ for each possibility, divide n by the corresponding integer. if it divides, accept. otherwise set $i = i + 1$, repeat step 2.