4/22/2019

Recap:

* Ex. of Bool. var.

$$\phi = (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_2 \vee x_3)$$

(0,0,1) satisfies Boolean equation above.

* Define SAT = $\{\phi \mid \phi$ = satisfiable Boolean formula$\}$.

* Prop: SAT $\in$ NP

Reducable:

Say lang A is polynomial red. to lang. B if $\exists$ TM M s.t. M always terminates in poly. time & input A $\iff$ input B.

* Prop: Let A be poly. reduc. to B.

1) B $\in$ P $\Rightarrow$ A $\in$ P

2) B $\in$ NP $\Rightarrow$ A $\in$ NP

Pf:

If N is TM (non-deterministic) that decides B, then run M & run N on output of M.

Def: lang $A \subset \{0,1\}^*$ is NP-complete if $A \in NP$ & $A \in P \Longleftrightarrow P = NP$

Prop: If A is ~~PA~~ poly. time reduc. to B & $B \in NP$, then
A NP-complete $\Longrightarrow$ B - NP complete

Pf: Let M be TM poly reduc. A to B.
Since A is NP complete,
every lang. $C \in NP$ can be red. to A in poly time.

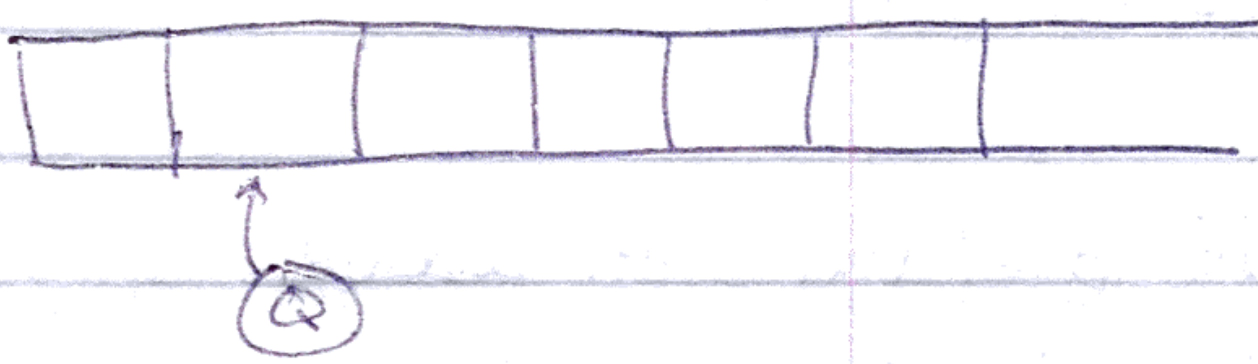
Since A can be reduc. to B in poly time
$\Rightarrow$ every lang. $C \in NP$ can be red. to B $\Rightarrow$ B NP-complete.

Cook-Levin: SAT = NP-complete
IDEA: Given a lang $A \in NP$,
WTS it is reduc. to SAT in poly. time.
Sps M is NDTM decides A in poly. time

GOAL: Cstruct Bool. exp. ϕ from M & input w, M accepts $w \Longleftrightarrow$
ϕ is sat

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$.

Note # of sq.'s TM uses $\leq p(n)$.



Notation:

$Q = \{q_0, q_1, q_2, \ldots, q_r, \cancel{q_{accept}}, q_{r+1}, q_{r+2}\}$.

$\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_s\}$.

$w = w_1, \ldots, w_n$ be input, $w_i \in \{0,1\}$.

$\delta : Q \times \Gamma \longrightarrow P(Q \times \Gamma \times \{L, R\})$

$\delta(q, \gamma) \subset Q \times \Gamma \times \{L, R\}$.

Goal:

From M & w, (of length n), construct ϕ (poly. length) where ϕ satisfiable $\Longleftrightarrow$ M accepts w.

Arrangeable

Let's assume that $|\delta(q,\gamma)| = 2$.

~~(Arr~~

Let $\delta(q,\gamma) = \{\delta(q,\gamma)^{(0)}, \delta(q,\gamma)^{(1)}\}$.

$$\delta(q,\gamma)^{(0)} = (\delta(q,\gamma)_Q^{(0)}, \delta(q,\gamma)_\Gamma^{(0)}, \delta(q,\gamma)_H^{(0)})$$
$$\delta(q,\gamma)^{(1)} = \text{-----------}$$

INTRODUCING Bool Var's for above:

$Q_{i,k}$ = (At time i, M is in state q_k)
$H_{i,j}$ = At time i, j^{th} space has the head
$S_{i,j,l}$ = At time i, sq. j contains γ_l

*Note: sq. = square.

Let's study the constraint of the variables.

① Initial condition:

$Q_{1,0} = 1 \quad Q_{1,k} = 0$ w/ $k \neq 0$
$H_{1,1} = 1 \quad H_{1,j} = 0$ w/ $j = 2, \ldots, p(n)$
$S_{1,j,l} = 1 \iff w_j = \gamma_l$

*Note: w/ = with.

② Condition at time i

(This is where we define how TM can move).

$\phi_{\text{constraint}}$

$$Q_{i+1,k} = 1 \iff \left(\delta(q,\gamma)^{(0)}_{Q} = q_k \;\&\; x_i = 0\right) \vee \left(\delta(q,\gamma)^{(1)}_{Q} = q_k \;\&\; x_i = 1\right)$$

$$\cancel{H_{i+1}}\; H_{i+1,j} = 1 \iff \left(H_{i,j-1} = 1 \;\&\; \delta(q,\gamma)^{(0)}_{H} = \{R\} \;\&\; x_i = 0\right)$$
$$\vee \left(H_{i,j+1} = 1 \;\&\; \delta(q,\gamma)^{(0)}_{H} = \{L\} \;\&\; x_i = 0\right)$$
$$\vee \left(H_{i,j-1} = 1 \;\&\; \delta(q,\gamma)^{(1)}_{H} = \{R\} \;\&\; x_i = 1\right)$$
$$\vee \cdots\cdots$$

$$S_{i+1,j,l} = 1 \iff \delta(q,\gamma)^{(0)} = \gamma_l \;\&\; x_i = 0$$

③ Final Condition: (Define $q_{r+1} := q_{\text{accept}}$)

Accepts at time i if $Q_{i,r+1} = 1$

$$\phi_{\text{final}}\; \cancel{\phi_{\text{final}}} = \bigvee_{i=1}^{p(n)} Q_{i,r+1}$$

$$\Longrightarrow \phi := \phi_{\text{initial}} \wedge \phi_{\text{constraint}} \wedge \phi_{\text{final}}$$

ϕ satisfiable $\iff$ M accepts w.