Math 403B: Introduction to Modern Algebra, Winter Quarter 2018 Jarod Alper Homework 2 Due: Monday, January 22

**Problem 2.1.** Judson 16.6.1

**Problem 2.2.** Judson 16.6.3

**Problem 2.3.** Suppose R is a ring with exactly t distinct unit elements.

- (a) Prove that every unit u satisfies  $u^t = 1$ .
- (b) Deduce Euler's theorem: if n and a are relatively prime positive integers, then  $a^{\varphi(n)} \equiv 1 \mod n$

where  $\varphi(n)$  is the number of relatively positive integers less than n relatively prime to n.

Problem 2.4. Goodman 1.11.9

Problem 2.5. Goodman 1.11.11

**Problem 2.6.** Judson 16.6.4

**Problem 2.7.** Judson 16.6.5

**Problem 2.8.** Judson 16.6.7

**Problem 2.9.** Judson 16.6.8

Problem 2.10. Judson 16.6.9

Problem 2.11. Judson 16.6.11

**Problem 2.12.** Find all the units in  $\mathbb{Z}[i]$ .