Math 340: Advanced Linear Algebra
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Sample exam problems

Problem 1.1. Let V be a vector space over $\mathbb{F}$ and $W_1, W_2 \subset V$ be subspaces such that $W_1 \subset W_2$. Show that there is an onto linear transformation $T\colon V/W_1 \to V/W_2$.

Problem 1.2. Recall that $P_i(\mathbb{R})$ denotes the vector space of polynomials of degree $\leq i$ with real coefficients. Let $S = \{x^2+1, x^3+x, x^4+x^2, x^4-1\} \subset P_4(\mathbb{R})$. What is the dimension of the span of S?

Problem 1.3. Consider the linear transformation $T\colon P_4(\mathbb{R}) \to P_4(\mathbb{R})$ defined by $T(f) = f - f'$. Consider also the basis $\beta = \{1, x, x^2, x^3, x^4\}$ of $P_4(\mathbb{R})$. What is the matrix representation $[T]_\beta$ of T with respect to the basis β? Show that T is invertible.

Problem 1.4. Let $A = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$. What are the eigenvalues of A? What are the corresponding eigenspaces? Is A diagonalizable?

Problem 1.5. Show that if A and B are similar matrices, then A and B have the same characteristic polynomial.

Problem 1.6. Consider $\mathbb{R}^3$ with the standard inner product. Let a, b, c be real numbers which are not all zero. Let $W \subset \mathbb{R}^3$ be the plane of points $(x, y, z) \in \mathbb{R}^3$ defined by the equation $ax + by + cz = 0$. What is the orthogonal complement $W^\perp$?

Problem 1.7. Consider the linear transformation $T\colon \mathbb{C}^2 \to \mathbb{C}^2$ defined by $T(x, y) = (x - iy, ix + y)$. Is T normal? If so, find an orthonormal basis consisting of eigenvectors of T.

Problem 1.8. Consider $\mathbb{C}^4$ with the standard inner product. Find an orthonormal basis for the span of $\{(1,0,1,0), (0,1,1,0), (1,1,1,1)\}$.

Problem 1.9. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. Is A a diagonalizable matrix over the real numbers? If so, find a real $n \times n$ matrix P such that $P^{-1}AP$ is diagonal.

Problem 1.10. . Recall that if $A = (A_{i,j})$ is an $n \times n$ matrix, then the trace of A is defined as $\operatorname{tr}(A) = \sum_{i=1}^n A_{i,i}$. Show that if A and B are $n \times n$ matrices, then $\operatorname{tr}(AB) = \operatorname{tr}(BA)$.