Algebra 2, Semester 1 2015
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Tutorial 5
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Tutorial 5.1. Determine the splitting fields $\mathbb{Q} \subseteq K$ of the following polynomials defined over $\mathbb{Q}$ and compute the degree $|K : \mathbb{Q}|$.
(a) $f(x) = x^3 - 2$.
(b) $f(x) = (x^2 - 3)(x^3 + 1) \in \mathbb{Q}[x]$.

Tutorial 5.2. Let η be a primitive 9th root of unity.
(a) What is the minimal polynomial for η?
(b) Express η^{-1} as a $\mathbb{Q}$-linear combination of $1, \eta, \eta^2, \ldots, \eta^5$.

Tutorial 5.3. Show that the multiplicative group $\mathbb{F}_{11}^{\times}$ is isomorphic to $\mathbb{Z}/10\mathbb{Z}$.

Tutorial 5.4. Let p be a prime.
(a) Show that for any integer $1 < i < p$, then the prime p divides the binomial coefficient $\binom{p}{i}$.
(b) Conclude that if K is a field of characteristic p, then there is an equality
$$(x - a)^p = x^p - a^p.$$

Let $K \subseteq L$ be a field extension. Recall that we say $\alpha \in L$ is *separable over* K if the minimal polynomial of α over L has no multiple roots. We say that $K \subseteq L$ is a *separable* field extension if every element $\alpha \in L$ is separable over K. You may use freely the following two properties (which will be proved next week):

- If the characteristic of K is zero, then $K \subseteq L$ is separable.
- If the characteristic of K is p and every element of K has a pth root, then $K \subseteq L$ is separable.

Tutorial 5.5. For each of these field extensions, determine (a) whether it is normal and (b) whether it is separable.
(a) $\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{3})$.
(b) $\mathbb{F}_2 \subseteq \mathbb{F}_2[x]/(x^3 + x + 1)$.
(c) $\mathbb{F}_p(t) \subseteq \mathbb{F}_p(t)[x]/(x^p - t)$ where p is a prime.