Algebra 2, Semester 1 2015
Jarod Alper
Tutorial 4
Friday, March 13

Tutorial 4.1. Let $L = \mathbb{Q}(\sqrt{2}, i) \subset \mathbb{C}$. Find all intermediate field extensions

$$\mathbb{Q} \subset K \subset L.$$

Tutorial 4.2. Determine (with proof) the degrees of the following field extensions, and write down an explicit basis for each:

(a) $\mathbb{Q}(\sqrt{3}, i)/\mathbb{Q}$
(b) $\mathbb{Q}(e^{2\pi i/6})/\mathbb{Q}$ for a prime p
(c) $\mathbb{Q}(\sqrt{5+2\sqrt{6}}, \sqrt{6})/\mathbb{Q}$
(d) $\mathbb{Q}(\sqrt[3]{\pi})/\mathbb{Q}(\pi)$

Tutorial 4.3. Let $K \subset L$ be a field extension of prime degree. Prove that L/K is primitive.

Tutorial 4.4.

(1) Show that a square can be duplicated using only a ruler and a compass.
(2) Construct a square using only a ruler and a compass.
(3) Construct a hexagon using only a ruler and a compass.
(4) Construct a regular pentagon using only a ruler and a compass.
Hint: Consider formulas for $\cos(5\theta)$.