Algebra 2, Semester 1 2015
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Tutorial 1

Tutorial 1.1. Using the method discussed in class to find the solution to:

$$x^3 - 15x - 4 = 0.$$

Which solutions are real? rational?

Tutorial 1.2. Let $f \in k[\alpha_1, \ldots, \alpha_n]$ be any polynomial. Let $f_1, \ldots, f_k$ be the orbit of f under the action of S_n. (Notice that one of the f_i is equal to f.) Then

(a) Show that $f_1 + \cdots + f_k$ is symmetric.
(b) If $s(x_1, \ldots, x_k)$ is any symmetric polynomial in $x_1, \ldots, x_k$, show that $s(f_1, \ldots, f_k)$ is a symmetric polynomial in $\alpha_1, \ldots, \alpha_n$.

Tutorial 1.3. Express

$$x_1^2x_2 + x_1^2x_3 + x_1x_2^2 + x_1x_3^2 + x_2^2x_3 + x_2x_3^2$$

as a polynomial in s_1, s_2, s_3.

Tutorial 1.4. Let

$$\begin{aligned} f_1 &= (\alpha_1 + \alpha_2)(\alpha_3 + \alpha_4) \\ f_2 &= (\alpha_1 + \alpha_3)(\alpha_2 + \alpha_4) \\ f_3 &= (\alpha_1 + \alpha_4)(\alpha_2 + \alpha_3) \end{aligned}$$

(a) Express $f_1 + f_2 + f_3$ as a polynomial in $s_1, \ldots, s_4$.
(b) Express $f_1f_2 + f_1f_3 + f_2f_3$ as a polynomial in $s_1, \ldots, s_4$.
(c) Express $f_1f_2f_3$ as a polynomial in $s_1, \ldots, s_4$.

Tutorial 1.5. Recall that if $h(x_1, \ldots, x_n)$ is a polynomial, then $L(h)$ denotes the lowest term of h with respect to the lexigraphical ordering. Is it true that

$$L(fg) = L(f)L(g)$$

for polynomials f and g.