Algebra 2, Semester 1 2015
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Homework 4
Due: Monday, March 16

Problem 4.1.

(a) How many irreducible polynomials $f(x) \in \mathbb{F}_2[x]$ are there of degree 3?
(b) What about degree 4?
(c) Show explicitly that $\mathbb{F}_2[x]/(x^3+x+1)$ and $\mathbb{F}_2[x]/(x^3+x^2+1)$ are isomorphic.

Problem 4.2. Let $L = \mathbb{Q}(\sqrt{2}, \sqrt{3}) \subset \mathbb{C}$. Find all intermediate field extensions

$$\mathbb{Q} \subset K \subset L.$$

Problem 4.3. Determine (with proof) the degrees of the following field extensions, and write down an explicit basis for each:

(a) $\mathbb{Q}(1+\sqrt[3]{2}+\sqrt[3]{4})/\mathbb{Q}$
(b) $\mathbb{Q}(e^{2\pi i/p})/\mathbb{Q}$ for a prime p
(c) $\mathbb{Q}(\sqrt{10+4\sqrt{6}}, \sqrt{6})/\mathbb{Q}$

Problem 4.4 (Extra credit)**.** Let p be a prime number and K be any field of characteristic p; that is, for any $x \in K$,

$$p \cdot x = \underbrace{x + \cdots + x}_{p \text{ times}} = 0.$$

Prove that for any $a \in K$, the polynomial $x^p - a \in K[x]$ is either irreducible or has a root in K. You may assume that there exists a field extension $K \subseteq L$ such that $x^p - a$ has a root in L.

Problem 4.5.

(a) Show that any angle can be bisected by ruler and compasses.
(b) Show that the angle $90°$ can be trisected by ruler and compasses.
(c) *Extra credit:* Show that the angle θ can be trisected by ruler and compasses if and only if the polynomial

$$4x^3 - 3x - \cos\theta$$

is reducible over $\mathbb{Q}(\cos\theta)$.