Algebra 2, Semester 1 2015
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Homework 2
Due: Monday, March 2

Problem 2.1. In 1545, Cardan was given the following problem: Divide 10 into three parts such that they shall be in continued proportion and the product of the first two is 6.[1]

Use Lagrange's method to solve the above classical problem.

Problem 2.2. Let A be a ring and $I \subset A$ be an ideal.

(a) Prove that $I \subset A$ is prime if and only if A/I is an integral domain.
(b) Prove that $I \subset A$ is maximal if and only if A/I is a field.
(c) Conclude that every maximal ideal is prime.

Problem 2.3.

(a) Show that $\mathbb{R}[x]/(x^2 + x + 1) \cong \mathbb{C}$.
(b) How many elements does the quotient ring $\mathbb{F}_3[x, y]/(x^3, xy, y^3)$ have?

Problem 2.4. Recall that the Dihedral group D_8 is the group of size 8 with generators x and y such that $x^4 = y^2 = \mathrm{id}$ and $yx = x^{-1}y$. If $f = (\alpha_1 + \alpha_2)(\alpha_3 + \alpha_4)$, show that the subgroup

$$H = \{\sigma \in S_4 \mid \sigma \cdot f = f\}$$

is isomorphic to D_8.

Problem 2.5. Use a similar method of Lagrange's solution of the quartic to solve the cubic equation.

Hint: Search for a polynomial $f(\alpha_1, \alpha_2, \alpha_3)$ whose orbit under S_3 consists of only 2 elements.

[1] Cardan states it as:

> Exemplum. Fac ex 10 tres partes proportionales, ex quarum ductu primæin secundam, producantur 6. Hanc proponebat Ioannes Colla, & dicebat solui non posse, ego uero dicebam, eam posse solui, modum tame ignorabam, donec Ferrarius eum inuenit." *Ars Magna* cap. XXXIV, qvæstio V; 1545 ed., fol. 73, v.

See pg.467 *History of Mathematics, Band 2* by David Eugene Smith for an account of the history of this problem.