Practice questions for the midterm

1. Suppose that G is an abelian group and that $a,\ b \in G$. Suppose that $|a| = 3$ and $|b| = 5$. Prove that $|ab| = 15$.

2. Suppose that G is a group and that $c \in G$. Suppose that $|c| = 15$. Prove that there exist elements $a,\ b \in G$ such that $|a| = 3$, $|b| = 5$, and $ab = c$.

3. Let $G = S_8$. Show that there exist elements $a,\ b \in G$ such that $|a| = 3$ and $|b| = 5$, but $|ab| \neq 15$.

4. Let σ be the following element in S_9:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 8 & 9 & 7 & 6 \end{pmatrix} .$$

(a) Find the cycle decomposition of σ.

(b) Let $H = \langle \sigma \rangle$, the cyclic subgroup of S_9 generated by σ. Determine $|H|$.

(c) Does there exist an element $\tau \in S_9$ such that $\tau\sigma\tau^{-1} = \tau^3$? If so, find such a τ. If not, explain why.

(d) Does there exist an element $\tau \in S_9$ such that $\tau\sigma\tau^{-1} = \tau^2$? If so, find such a τ. If not, explain why.

5. Give an example of a nonabelian group G of order 42.

6. Give two examples of non-isomorphic groups G such that G is nonabelian, but every proper subgroup of G is cyclic.

7. Give an example of non-isomorphic groups G such that G is nonabelian, every proper subgroup of G is abelian, and at least one proper subgroup is not cyclic.

8. Determine the center of the group Q_8. Determine the center of the group D_4. Determine the center of the group $G = A \times B$, where A and B are groups of order 4.