The Two Phase Simplex Algorithm

Solve the following LPs using the two phase simplex algorithm.

1.
$$
\begin{array}{rrcrcl}
\text{maximize} & 3x_1 & + & x_2 & & \\
\text{subject to} & x_1 & - & x_2 & \leq & -1 \\
& -x_1 & - & x_2 & \leq & -3 \\
& 2x_1 & + & x_2 & \leq & 4 \\
& 0 & \leq & x_1, & x_2 &
\end{array}
$$

Solution: $(1, 2)$

2.
$$
\begin{array}{rrcrcrcl}
\text{maximize} & & & 3x_2 & + & x_3 & & \\
\text{subject to} & x_1 & + & 2x_2 & + & x_3 & \leq & 2 \\
& 2x_1 & + & x_2 & - & x_3 & \leq & -1 \\
& 3x_1 & + & 2x_2 & + & x_3 & \leq & 3 \\
& 0 & \leq & x_1, & x_2, & x_3 &
\end{array}
$$

Solution: $(0, 1/3, 4/3)$

3.
$$
\begin{array}{rrcrcrcl}
\text{maximize} & -2x_1 & - & 2x_2 & - & x_3 & & \\
\text{subject to} & x_1 & - & 2x_2 & + & x_3 & \leq & -2 \\
& 2x_1 & & & - & 2x_3 & \leq & 0 \\
& -x_1 & + & x_2 & & & \leq & -1 \\
& 0 & \leq & x_1, & x_2, & x_3. &
\end{array}
$$

Solution: infeasible

4.
$$
\begin{array}{rrcrcrcl}
\text{maximize} & x_1 & + & x_2 & + & x_3 & & \\
\text{subject to} & -2x_1 & + & 3x_2 & + & x_3 & \leq & -1 \\
& x_1 & + & 3x_2 & - & x_3 & \leq & -1 \\
& 2x_1 & - & x_2 & - & 3x_3 & \leq & -6 \\
& 0 & \leq & x_1, & x_2, & x_3. &
\end{array}
$$

Solution: This problem is feasible, but unbounded.

5.
$$
\begin{array}{rrcrcrcrcl}
\text{maximize} & -x_1 & + & x_2 & + & x_3 & + & 4x_4 & & \\
\text{subject to} & 2x_1 & + & x_2 & & & + & x_4 & \leq & 3 \\
& -2x_1 & - & x_2 & + & x_3 & + & 4x_4 & \leq & -2 \\
& 2x_1 & - & 2x_2 & + & x_3 & + & x_4 & \leq & -4 \\
& 0 & \leq & x_1, & x_2, & x_3, & x_4. &
\end{array}
$$

Solution: $(0, 3, 1, 0)$

6.

$$\text{maximize} \quad 2x_1 \quad - \quad x_3 \quad + \quad x_4$$

$$\begin{array}{rrrrrrrrr}
\text{subject to} & x_1 & + & x_2 & & & & - & x_4 & \leq & 7 \\
& x_1 & - & x_2 & + & x_3 & + & x_4 & \leq & -2 \\
& & 0 & \leq & x_1, & x_2, & x_3, & x_4. &
\end{array}$$

Solution: The LP is unbounded.

7.

$$\text{maximize} \quad x_1 \quad + \quad x_2 \quad + \quad x_3 \quad + \quad x_4$$

$$\begin{array}{rrrrrrrrr}
\text{subject to} & -x_1 & + & 2x_2 & + & x_3 & + & x_4 & \leq & 3 \\
& x_1 & - & 2x_2 & - & x_3 & + & 2x_4 & \leq & -1 \\
& 2x_1 & + & x_2 & - & x_3 & & & \leq & -2 \\
& & 0 & \leq & x_1, & x_2, & x_3, & x_4. &
\end{array}$$

Solution: $(1, 0, 4, 0)$.