Graphical solutions of two dimensional LPs

Solution Procedure

Step 1: Graph each of the linear constraints indication on which side of the constraint the feasible region must lie. Don't forget the implicit constraints!

Step 2: Shade in the feasible region.

Step 3: Draw the gradient vector of the objective function.

Step 4: Place a straightedge perpendicular to the gradient vector and move the straightedge either in the direction of the gradient vector for maximization, or in the opposite direction of the gradient vector for minimization to the last point for which the straightedge intersects the feasible region. The set of points of intersection between the straightedge and the feasible region is the set of solutions to the LP.

Step 5: Compute the exact optimal vertex solutions to the LP as the points of intersection of the lines on the boundary of the feasible region indicated in Step 4. Then compute the resulting optimal value associated with these points.

1. Sketch the graph of the constraint region for the following LP's. Then solve them by sketching the optimal level set of the objective function.

   (a)

$$\begin{array}{lrcrcr} \text{maximize} & 2x & + & 3y & & \\ \text{subject to} & -3x & + & y & \leq & 2 \\ & 4x & + & 2y & \leq & 44 \\ & 4x & - & y & \leq & 20 \\ & -x & + & 2y & \leq & 14 \\ & 0 & \leq & x, & y & \end{array}$$

   (Hint: optimal value = 42)

   (b)

$$\begin{array}{lrcrcr} \text{minimize} & x & + & y & & \\ \text{subject to} & -x & + & y & \leq & 3 \\ & 2x & + & y & \leq & 18 \\ & & & y & \geq & 6 \\ & 0 & \leq & x, & y & \end{array}$$

   (Hint: optimal value = 9)

   (c)

$$\begin{array}{lrcrcr} \text{maximize} & & 3x & + & 2y & \\ \text{subject to} & x & + & 4y & \leq & 16 \\ & x & - & 2y & \leq & -1 \\ & 1 & \leq & y & \leq & 4 \\ & 0 & \leq & x & \leq & 4 \end{array}$$

   (Hint: optimal value = 18)

2. Graph the following function of α by graphically solving the necessary LPs:

$$
\begin{array}{rlrcrcr}
v(\alpha) := & \text{maximize} & x_1 & + & \alpha x_2 & & \\
& \text{subject to} & x_1 & - & x_2 & \leq & 4 \\
& & x_1 & + & x_2 & \leq & 6 \\
& & -3x_1 & + & x_2 & \leq & -6 \\
& & 0 \leq & x_1, & x_2 & . &
\end{array}
$$

(Hint: $v(\alpha) = 4$ for all $\alpha \leq -1$)