Seattle Noncommutative Algebra Day

March 17-18, 2023

ABSTRACT

Automorphisms of the quantum grassmannian

Tom Lenagan

University of Edinburgh

The automorphism group of the generic quantum grassmannian is calculated. The main tool is the notion of noncommutative dehomogenisation. This reduces the problem to that of finding the automorphism group of quantum matrices and this problem has already been solved. The talk will concentrate on small examples to exemplify the main ideas and will avoid technical proofs.

Homological Regularities

Ellen Kirkman

Wake Forest University

Let A be a noetherian connected graded $\Bbbk$-algebra with a balanced dualizing complex, and let X be a cochain complex of graded left A-modules. The elements of X possess both an internal and various homological degrees, and it is useful to study the relationships between these degrees. Jørgensen and Dong-Wu extended the study of Tor-regularity and Castelnuovo-Mumford regularity from commutative algebras to noncommutative algebras. We consider these regularities further, and define new numerical invariants that involve linear combinations of internal and homological degrees. This is joint work with Robert Won and James J. Zhang.

Actions of the quantum double of certain finite groups on quadratic AS-regular algebras

Frank Moore

Wake Forest University

(Joint work with Ellen Kirkman and Tolulope Oke) The quantum double $D(H)$ of a Hopf algebra H was originally introduced by Drinfel'd in his study of solutions to the quantum Yang-Baxter equation. We use Witherspoon's calculation of the representation ring of the quantum double $D(G)$ of a finite group G to determine families of inner-faithful representations of the quantum double of some generalized quaternion groups. We examine several such representations in detail, and use them to identify some families of quadratic AS-regular algebras (in fact, double Ore extensions) on which $D(G)$ acts. We compute some standard invariants for these algebras, as well as the homological determinant for these actions. Time permitting, we will also discuss generators for the ring of invariants.

Categories of hypergroups and hyperstructures

Manuel Reyes

University of California, Irvine

A hyperoperation on a set M is an operation that associates to each pair of elements a subset of M, thus generalizing a binary operation. Hypergroups and hyperrings are structures defined in terms of hyperoperations. While they were respectively defined in the 1930s and 1950s, they have recently become more prominent through various appearances in number theory, combinatorics, and absolute algebraic geometry. However, to date there has been relatively little attention given to categories of hyperstructures.

We will discuss several categories of hyperstructures, which we view as structures defined in the category of relations but whose morphisms are ordinary set maps. A common theme is that in order for these categories to enjoy good properties (such as (co)completeness), we must allow for the product or sum of two elements to be the empty subset, a condition that is typically forbidden for hypergroups. Most significantly, we will discuss a category containing canonical hypergroups that has a closed monoidal structure reminiscent of the tensor product of abelian groups. This is joint work with So Nakamura.

Twisting Manin's universal quantum groups and comodule algebras

Hongdi Huang

Rice University

In this talk, we will discuss the homological properties invariant under Morita-Takeuchi equivalence. In particular, we consider the infinite coaction of the Manin's universal quantum groups on an AS-regular algebra. As a consequence, the AS-regularity is invariant under 2-cocycle twist. This is joint work with Van C. Nguyen, Charlotte Ure, Kent B. Vashaw, Padmini Veerapen, Xingting Wang.

On the Twists of Graded Poisson Algebras

Xin Tang

Fayetteville State University

Various twists have been extensively studied in the context of noncommutative algebra. In this talk, we will present a Poission version of the twist for graded Poisson algebras and study its applications. In particular, we will show that every graded Poisson algebra structure on a connected graded polynomial ring is a twist of a unimodular Poisson algebra structure on this polynomial ring. We will define the notion of rigidity under graded twisting and calculate the rigidity for several families of graded Poisson algebras. A couple of results on Poisson cohomologies will be presented as well. This is joint work with Xingting Wang and James Zhang.

Poisson Valuation

Xingting Wang

Howard University

We will talk about Poisson valuation and its application in computing Poisson automorphism groups of Poisson elliptic algebras. It is joint work with Hongdi Huang, Xin Tang and James Zhang.

Ozone groups and centers of skew polynomial rings

Jason Gaddis

Miami University

The ozone group of a noncommutative algebra A is defined as the group of automorphisms of A which fix every element of its center. In this talk, I will discuss the ozone group in the context of PI skew polynomial rings. Using the ozone group, along with other invariants, gives a way to determine whether the center of a PI skew polynomial is Gorenstein (resp. regular) in low dimension. This is joint work with Kenneth Chan, Robert Won, and James J. Zhang.

Unimodular H-comodule algebras

Harshit Yadav

Rice University

Unimodular Hopf algebras H are an interesting subclass of Hopf algebras which can be used to construct invariants of 3-manifolds. For instance, semisimple Hopf algebras are unimodular. In this talk, inspired by applications to TQFTs, we will introduce the notion of unimodular H-comodule algebras. We will also provide examples and discuss some open questions.

Cocycle twists, Comodule Algebras, and certain Homological Invariants

Padmini Veerapen

Tennessee Tech University

In this talk, we will examine 2-cocycle twists in the context of comodule algebras. Moreover, we show that certain homological regularities (in the spirit of recent work by Kirkman, Zhang, and Won) remain invariant under 2-cocycle twists.

Symmetries captured by weak Hopf algebra actions

Robert Won

George Washington University

If A is a $\Bbbk$-algebra, then the group $\mathrm{Aut}(A)$ of algebra automorphisms of A can be viewed as the algebraic object which captures the symmetries of A. Likewise, the Lie algebra $\mathrm{Der}(A)$ of derivations of A can also be viewed as capturing the symmetries of A. We unite and generalize these notions by defining an object $\mathrm{Sym}_{\mathcal{C}}(A)$, which captures the symmetries of A by actions of objects in a category $\mathcal{C}$ consisting of Hopf-like objects. In particular, we show that if A is a non-connected algebra, then some of its symmetries are naturally captured by groupoids, Lie algebroids, and weak Hopf algebras.

This work is joint with Fabio Calderon, Hongdi Huang, and Elizabeth Wicks.

Twisted tensor products of bialgebras and Frobenius algebras

Amrei Oswald

University of Washington

We are interested in when the twisted tensor product of two bialgebras inherits a bialgebra structure in the case where the comultiplication is induced by the inverse of the twisting map. We show this only gives a bialgebra structure when the twisting map is trivial. However, we find that the twisted tensor product of two Frobenius algebras is always Frobenius. We also characterize when twisted tensor products of separable algebras are separable, and we prove that twisted tensor products of special Frobenius algebras are special Frobenius. All of the definitions and proofs are given diagrammatically, so these results hold in monoidal categories. This is joint work with Pablo S. Ocal.