Seattle Noncommutative Algebra Day

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ABSTRACT

Recurrence relations on scalars associated to homotopy lifting maps on Hochschild cohomology

Tolulope Oke

Wake Forest University

The Gerstenhaber bracket structure on Hochschild cohomology of algebras can be described and computed using homotopy lifting maps. Koszul quiver algebras have a resolution with a comultiplicative structure that can be computed using some algorithms. Introducing new scalars, we describe homotopy lifting maps associated to cocycles on Hochschild cohomology of Koszul quiver algebras using this resolution. We show that the scalars can be described by some recurrence relations and give some examples where these scalars appear in the literature.

Delooping levels versus finitistic dimensions

Birge Huisgen-Zimmerman

University of California at Santa Barbara

Recently, Vincent Gelinas introduced a new homological invariant of noetherian rings: the delooping level. Its strong relationship to depth and finitistic dimensions makes this invariant a promising tool towards a better understanding of the more traditional homological invariants. We will provide examples, review some of Gelinas's results, and answer the concluding question of his paper in the negative. Moreover, we will modestly enlarge the class of algebras for which the delooping level, and hence also the big finitistic dimension, is known to be finite.

Quantizing the maximal spectrum

Manuel Reyes

University of California, Irvine

Integrating perspectives from work of Takeuchi, Batchelor, Kontsevich-Soibelman, and Le Bruyn, we view the Sweedler dual coalgebra of an algebra with many finite-dimensional representations as a quantization of its maximal spectrum. We will explain a sense in which it is the minimal contravariant functor from algebras to coalgebras that performs as expected on finite-dimensional semisimple algebras, even though it yields a larger invariant than expected for commutative algebras. We will also discuss methods to compute the dual coalgebra in a variety of situations.

On the spectrum and support theory of a finite tensor category

Kent Vashaw

MIT

Determining the precise relationship between the cohomological support varieties of Quillen and Carlson on one hand and the tensor triangular support of Balmer on the other hand is a central problem in the study of stable categories of finite tensor categories. In this talk, we will introduce a certain subalgebra of the cohomology ring of a finite tensor category, and construct a surjective continuous map from the Balmer spectrum of the associated stable category to the Proj of this ring. We prove that this map is a homeomorphism in many cases, and conjecture that this holds in general. Concrete examples arising from the representation theory of Hopf algebras will be used to illustrate the general theory.

Drinfeld Hecke algebras and quantum polynomial rings

Anne Shepler

University of North Texas

We consider automorphisms of quantum polynomial rings (i.e., skew polynomial rings) from the viewpoint of a quantum determinant. This approach aids exploration of deformations of skew group algebras for reflection groups acting on quantum polynomial rings over fields of positive characteristic. In the nonquantum setting, a combinatorial approach is still helpful for studying deformations in positive characteristic, and we consider quadratic algebras built on relations that deform both the commutativity of the polynomial ring (Drinfeld-type) and the action of the group (Lusztig-type).

On non-counital Frobenius algebra

Harshit Yadav

Rice University

One characterization of Frobenius algebras is that they are finite-dimensional algebras A which come equipped with a coassociative, counital comultiplication map Δ that is an A-bimodule map. Here, we examine comultiplication maps for generalizations of Frobenius algebras: finite-dimensional self-injective (quasi-Frobenius) algebras. We show that large classes of such algebras, including finite-dimensional weak Hopf algebras, come equipped with a map Δ as above that is not necessarily counital. We also conjecture that this comultiplicative structure holds for self-injective algebras in general. This is joint work with Amanda Hernandez and Chelsea Walton.

Zhang twists and cocycle twists of Hopf algebras

Padmini Veerapen

Tennessee Tech University

In this talk, we will explore how a Zhang twist, that is, a twist of an algebra's multiplicative structure by an automorphism, can be extended to a twist of a Hopf algebra. We do so by twisting a bialgebra and then, lift it to a Hopf algebra using Takeuchi's Hopf envelope construction. Finally, we examine when our construction coincides with a 2-cocycle twist of the Hopf algebra.

Quantum symmetric pairs via star products

Milen Yakimov

Northeastern University

The systematic study of quantum symmetric pairs (QSPs) was initiated by Gail Letzter in 1999. The area has been greatly developed in recent years. We will present a new approach to the theory of quantum symmetric pairs for symmetrizable Kac-Moody algebras based on star products on noncommutative graded algebras. It will be used to give solutions to two main problems in the area: (1) determine the defining relations of QSPs and (2) find a Drinfeld type formula for universal K-matrices as sums of tensor products over dual bases. This is a joint work with Stefan Kolb.

Twisting of graded quantum groups and solutions to the quantum Yang-Baxter equation

Xingting Wang

Howard University

Let H be a Hopf algebra over a field k such that H is $\mathbb{Z}$-graded as an algebra. In this talk, we introduce the notion of a twisting pair for H and show that the Zhang twist of H by such a pair can be realized as a 2-cocycle twist. We use twisting pairs to describe twists of Manin's universal quantum groups associated to quadratic algebras. Furthermore, we discuss a strategy to twist a solution to the quantum Yang-Baxter equation via the Faddeev-Reshetikhin-Takhtajan construction. If time permits, we illustrate this result for the quantized coordinate rings of $GL_n(k)$. This is joint work of H. Huang, V. Nguyen, C. Ure, K. Vashaw and P. Veerapen.

Poisson catenarity in Poisson nilpotent algebras

Ken Goodearl

University of California at Santa Barbara

Many Poisson algebras appear as semiclassical limits of quantized coordinate rings, and these Poisson algebras are expected to display properties that parallel those of the corresponding quantized coordinate rings, just as the latter are expected to display properties that parallel those of their classical counterparts. We will focus on the important property of *catenarity*. In the quantum and classical cases, this condition asks that all saturated chains of prime ideals between any two fixed primes have the same length; the parallel in the Poisson setting is *Poisson catenarity*, which asks for the analogous chain property in the poset of Poisson-prime ideals.

Catenarity is known to hold in classical coordinate rings of affine varieties and has been proved for many quantized coordinate rings. The parallel Poisson catenarity has recently been established by Launois and the speaker for the large class of Poisson algebras called *Poisson nilpotent*. On the other hand, it does *not* hold for *all* affine Poisson algebras. [Joint work with Stephane Launois]

Derived categories adapted to Koszul Calculus

Roland Berger

Université Jean-Monnet (Saint-Etienne)

In the context of quiver algebras A having homogeneous quadratic relations, Rachel Taillefer and I defined Koszul complex Calabi-Yau algebras and proved a Poincaré Van den Bergh duality theorem for these algebras (JLMS 2020). However, in absence of further structure, the duality theorem is not expressed as a cap-product by a fundamental class. In my talk, I present an enriched structure on the Koszul complex $K(A)$ permitting to realize such a cap-product. This is given by a dg algebra $\tilde{A}$ such that $K(A)$ is a dg $\tilde{A}$-bimodule compatible with its structure of A-bimodule. So it is natural to work in the derived category of dg $\tilde{A}$-bimodules in A-Bimod, following the formalism of Yekutieli's book Derived Categories (CUP 2020). In this derived category, I'll discuss the question to derive the functors involved in the duality theorem.

Quotients of the Associative Algebra Operad of GK-dimension 5

Yanhong Bao

Anhui University

Let $\mathcal{A}ss$ denote the associative algebra operad that encodes the category of unital associative algebras. Quotient operads of $\mathcal{A}ss$ relate to polynomial identity algebras (PI-algebras) closely. In fact, a PI-algebra is equivalent to an algebra over $\mathcal{A}ss/\mathcal{I}$ for some nonzero operadic ideal of $\mathcal{A}ss/\mathcal{I}$. In this talk, we will introduce the classification of quotient operads $\mathcal{A}ss/\mathcal{I}$ of GK-dimension 5.

Homological Regularities

Ellen Kirkman

Wake Forest University

Let A be a noetherian connected graded $\Bbbk$-algebra with a balanced dualizing complex, and let X be a cochain complex of graded left A-modules. The elements of X possess both an internal and various homological degrees, and it is useful to study the relationships between these degrees. An example of a relation between a homological degree i and an internal degree j occurs when A is Koszul; then $\operatorname{Tor}_i^A(\Bbbk, \Bbbk)_j = 0$ when $i \neq j$, or the Tor-regularity of $\Bbbk$ is 0. Jørgensen and Dong-Wu extended the study of Tor-regularity and Castelnuovo-Mumford regularity from commutative algebras to noncommutative algebras. We consider these regularities further, and define new numerical invariants that involve linear combinations of internal and homological degrees. This is joint work with Robert Won and James J. Zhang.

Mixed bimodules of algebras over different operads

Cody Tipton

University of Washington

We will talk about a new structure called relative operads, that is in some ways an extension of the ideas of an operad, except the algebras over them are left modules, right modules, or bimodules. We will give plenty of examples of these and show that a specific class of these objects come from bimodules of operads.

Constructive Invariant Theory in the Exterior Algebra

Francesca Gandini

Kalamazoo College

In commutative invariant theory, Derksen showed that the generators of the Hilbert ideal can be found via elimination theory from the vanishing ideal of a subspace arrangement. We show that the same approach works over the exterior algebra and prove Noether's Degree Bound in this context. We also show a transference of bounds from the symmetric algebra to the exterior algebra. This approach also bounds some square-free invariants in the (-1)-skew algebra and motivates further investigations in the theory of skew polarization.

On odd-dimensional modular tensor categories

Julia Plavnik

Indiana University

Modular categories arise naturally in many areas of mathematics, such as conformal field theory, representations of braid groups, quantum groups, and Hopf algebras, low dimensional topology, and they have important applications in condensed matter physics.

One interesting class of modular categories is the one of odd-dimensional ones, which is closely related to maximally non-self dual (MNSD) modular tensor categories. In this talk, we will present some properties of odd-dimensional and MNSD modular categories and we will show some advances in their classification by rank.