

# Seattle Noncommutative Algebra Day

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## ABSTRACT

The Hochschild cohomology of gentle algebras

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Homological methods provide important information about the structure of associative algebras, revealing sometimes hidden connections amongst them. The Hochschild homology, cohomology -together with its graded algebra structure and its Gerstenhaber structure - of unital associative algebras over a field are invariants preserved by derived equivalences. The Lie algebra of derivations modulo inner derivations -that is, the first Hochschild cohomology of the algebra- is particularly interesting.

The family of gentle algebras has attracted the attention of many authors in recent times. These algebras were introduced by Assem and Skowroński in the '80s as a generalization of iterated tilted algebras of type  $A_n$ , and affine type  $\tilde{A}_n$ . They are connected to many other areas of mathematics such as dimer models, Lie algebras, cluster theory, and homological mirror symmetry.

We will give a complete description of the structure of the Hochschild cohomology ring of a gentle algebra  $A$  both as a graded commutative algebra and as Gerstenhaber algebra. Furthermore, we will show how these structures are encoded in the geometric surface model of the bounded derived category associated to a gentle algebra via its ribbon graph. Moreover, we will explain how the first Hochschild cohomology acts on the category of  $A$ -modules and on its derived category.

This is a joint work with Cristian Chaparro Acosta, Sibylle Schroll and Mariano Suárez-Álvarez.

# Rigidity, Residues and Duality: Overview and Recent Progress

**Amnon Yekutieli**

Ben Gurion University

In this lecture we explain the theory of rigid residue complexes in commutative algebra and algebraic geometry. Unlike all previous approaches to Grothendieck Duality, the rigid approach concentrates on rigid residue complexes over rings, and their intricate yet robust properties. Most of the lecture will be about the results for rings. The geometrization, i.e. the passage to rigid residue complexes on schemes and Deligne-Mumford (DM) stacks, by gluing, is fairly easy. In the geometric part of the theory, the main results are the Rigid Residue Theorem and the Rigid Duality Theorem for proper maps between schemes, and for tame proper maps between DM stacks. These results will only be outlined briefly.

More details are available in the eprint with the same title at

<https://arxiv.org/abs/2102.00255>

The lecture notes can be downloaded from

<http://www.math.bgu.ac.il/~amyekut/lectures/RRD-2021/notes.pdf>

## Building resolutions from Koszul complex

**Van Nguyen**

United States Naval Academy

For any local ring  $R$  with residue field  $\mathbb{k}$ , using the Koszul complex of  $R$  as building blocks, we construct a minimal free resolution of  $\mathbb{k}$  over  $R$ . We describe how the multiplicative structure and the Massey products of the homology algebra  $A$  of the Koszul complex are involved in the construction, and discuss several consequences of this construction.

## Cohen-Macaulay modules of noncommutative quadric hypersurfaces

**Jiwei He**

Hangzhou Normal University

Let  $S$  be a Koszul Artin-Schelter regular algebra, and let  $z \in S$  be a central regular element of degree 2. The quotient algebra  $A = S/Sz$  is usually called a noncommutative quadric hypersurface. In this talk, we will provide a method to verify whether a noncommutative quadric hypersurface is an isolated singularity, and some detailed computations of indecomposable maximal Cohen-Macaulay modules for some special noncommutative quadrics. We also introduce the notion of right pre-resolutions (quasi-resolutions) for noncommutative isolated singularities, which is a weaker version of quasi-resolutions introduced by Qin-Wang-Zhang. We show that right quasi-resolutions for noetherian bounded below and locally finite graded algebra with right injective dimension 2 are always Morita equivalent. For a noncommutative quadric hypersurface which is also a noncommutative isolated singularity, we show that it always admits a right pre-resolution. The talk is based on joint work with Yu Ye.

## The Zariski cancellation problem and the Abhyankar-Eakin-Heinzer theorem

**Jason Bell**

University of Waterloo

An algebra  $A$  is cancellative if whenever  $B$  is another algebra such that  $A[x] \cong B[x]$  we necessarily have  $A \cong B$ . Abhyankar, Eakin, and Heinzer showed that finitely generated commutative domains of Krull dimension one are cancellative. When one works in a noncommutative setting, it is more natural to work with Gelfand-Kirillov dimension, which is a noncommutative analogue of Krull dimension. Thus it is natural to ask whether the Abhyankar-Eakin-Heinzer theorem holds more generally for finitely generated domains of Gelfand-Kirillov dimension one. We show that this is the case in characteristic zero, but

somewhat surprisingly there are counterexamples in positive characteristic. This is joint work with Maryam Hamidizadeh, Hongdi Huang, and Helbert Venegas.

On pointed Hopf algebras with finite GK-dimension

**Nicolás Andruskiewitsch**

Universidad Nacional de Córdoba

TBA

Noncommutative Auslander Theorem for PI algebras

**Ruipeng Zhu**

Southern University of Science and Technology

Let  $R$  be a Noetherian PI connected graded Artin-Schelter regular algebra of GK-dimension  $d \geq 2$ , and  $H$  be a semisimple Hopf algebra acting on  $R$  inner faithfully and homogeneously. Suppose that the homological determinant is trivial. In this talk, we prove that, if the natural map

$$\varphi : R\#H \longrightarrow \text{End}_{R^H}(R)$$

is injective, then it is also surjective.

## Endomorphisms of Some Noncommutative Algebras

**Xin Tang**

Fayetteville State University

There has been an extensive study on the symmetries of algebras. Towards the study of algebra endomorphisms, there are mainly three types of tasks under consideration:

- (a) Classify all the bijective algebra endomorphisms of a given algebra;
- (b) Find a characterization of the bijective endomorphisms for a given algebra;
- (c) Identify algebras whose endomorphisms are all bijective.

In general, each task has proved to be extremely difficult with its long-standing problems or conjectures. In this talk, we will give a brief account of the vast literature and present new pieces of information with a focus on examples. A couple of questions will be discussed as well if time permits.

## Universal quantum semigroupoids

**Robert Won**

University of Washington

Weak bialgebras and weak Hopf algebras provide a context in which to study symmetries of not necessarily connected graded algebras. Here, we introduce the concept of a universal quantum linear semigroupoid (UQSGd), which is a weak bialgebra that coacts on a (not necessarily connected) graded algebra  $A$  universally while preserving grading. The UQSGd construction generalizes the universal quantum linear semigroups introduced by Manin in 1988, which are bialgebras that coact on a connected graded algebra universally while preserving grading. Our main result is that when  $A$  is the path algebra  $kQ$  of a finite quiver  $Q$ , the UQSGd is isomorphic to Hayashi's face algebra attached

to  $Q$ . This is joint work with Hongdi Huang, Chelsea Walton, and Elizabeth Wicks.

## Root of unity quantum cluster algebras and Cayley-Hamilton algebras

**Milen Yakimov**

Northeastern University

We will describe a theory of root of unity quantum cluster algebras, which covers as special cases the big quantum groups of De Concini, Kac and Procesi. All such algebras will be shown to be PI algebras. Inside each of them, we will construct a canonical central subalgebra which is proved to be isomorphic to the underlying cluster algebra. It is a far reaching generalization of the De Concini-Kac-Procesi central subalgebras in big quantum groups. We will prove that all root of unity quantum cluster algebras have two canonical structures of Cayley-Hamilton algebras in the sense of Procesi, one with respect to their full centers and one with respect to the constructed DKP central subalgebras. The Azumaya loci will be shown to contain the underlying cluster A varieties. This is a joint work with Thang Le (Georgia Tech) and Shengnan Huang (Northeastern Univ), and Bach Nguyen (Xavier Univ) and Kurt Trampel (Notre Dame Univ).

## When a weak bialgebra is a weak Hopf algebra

**Hongdi Huang**

Rice University

Weak bialgebras and weak Hopf algebras are generalized from bialgebras and Hopf algebras, respectively. Weak Hopf algebras admit a “nice” property that a direct sum of weak Hopf algebras forms a “big” weak Hopf algebra. They have been applied in many math branches recently. In this talk, we will investigate the existence of the antipode

of a weak bialgebra, i.e., understand when a weak bialgebra is a weak Hopf algebra.

## Filtered Frobenius algebras in monoidal categories

**Harshit Yadav**

Rice University

We develop filtered-graded techniques for algebras in monoidal categories with the main goal of establishing a categorical version of Bongale's 1967 result: A filtered deformation of a Frobenius algebra over a field is Frobenius as well. Towards the goal, we first construct a monoidal associated graded functor, building on prior works of Ardizzoni-Menini, of Galatius et al., and of Gwillian-Pavlov. Next, we produce equivalent conditions for an algebra in a rigid monoidal category to be Frobenius in terms of the existence of categorical Frobenius form; this builds on work of Fuchs-Stigner. These two results of independent interest are then used to achieve our goal. We illustrate these results by discussing braided Clifford algebras, which are filtered deformations of Bespalov et al.'s braided exterior algebras, and show that these are Frobenius algebras in symmetric rigid monoidal categories. This is joint work with Chelsea Walton.