

# How to Prepare Mathematics Homework

Homework assignments in mathematics serve a number of important functions:

1. They give the students practice with the technical details of the material they have learned in class, and help the students learn this material better.
2. They can be used as study tools for midterms, finals, and qualifying exams.
3. They give students practice at communicating mathematics. As a professional mathematician (either in academia or industry) one of the main roles of your job will be to explain mathematics to others.
4. They allow the professor to assess student performance.

Points 1,2, and 3 above are certainly the most important reasons for assigning homework. With these ideas in mind, I will describe criteria for preparing a well-written mathematics homework.

## Format of homework.

1. Make sure all solutions are turned in in order, pages are stapled, with sufficient margins so I can write comments.
2. Write out the statement of each problem before writing the solution.
3. Problem solutions should be written in paragraph format, not just a sequence of equations, and must consist of complete sentences.
4. Do not use the shorthand symbols  $\forall$   $\exists$   $\therefore$   $\because$ .
5. Please indicate on each problem who you worked with on the problem, or if you took ideas from another source. It is a violation of the university's academic integrity policy to copy the solution of a problem from another student, a book, a copy of solutions from another source, or elsewhere.

## Style of homework.

1. You should imagine that you are writing your homework solutions for another student in the class. Your solution should be able to convince another student that your solution is correct, without any oral explanation from you. Another student should be able to understand your solution completely via the written word, and whatever diagrams or tables you create in your solution.
2. The level of detail that I write on the board in class is usually not enough for a complete homework solution. Lectures use much shorthand that is inappropriate for a homework solution. Lectures often contain oral justifications of statements, which would need to be included as carefully written sentences in a homework.

3. Proof is the name of the game in advanced mathematics homework problems in pure mathematics. This includes problems that ask you a question of classification (e.g. “Determine all things that satisfy property X”). You must not only provide the classification, but prove that all things in your list satisfy property X and prove that there are no other things that satisfy property X.
4. Some proofs might involve a computation, possibly using a computer.
5. Assuming that the proofs satisfy the properties above, they should be as concise and to the point as possible. Long rambling proofs with extraneous partial results or unnecessary derivations may lose points.

### Specific Comments for Math 521 Homework

This course assumes familiarity with the material of Math 405 and 407. You are free to use knowledge of abstract algebra from previous course when working on the homework from Math 521.

### Example Homework Problem Solution

**Chapter 1 Problem 13:** Let  $p$  be a prime and  $a \in \mathbb{Z}_+$ . Show combinatorially that  $a^p - a$  is divisible by  $p$ .

I discussed this problem with John Q. Public and Jane Doe.

*Solution.* Let

$$S = [a]^p \setminus \{(n, n, \dots, n) : n \in [a]\}.$$

This set clearly has  $a^p - a$  elements. We will show that the cardinality of  $S$  is divisible by  $p$  by partitioning it into blocks, each of which has size  $p$ . Let the cyclic permutation group  $\langle (12 \cdots p) \rangle$  act on  $S$ , by cyclically permuting coordinates, that is:

$$(12 \cdots p) \cdot (n_1, \dots, n_p) = (n_2, n_3, \dots, n_p, n_1).$$

This action divides  $S$  into equivalence class: two elements of  $S$  are equivalent if they are cyclic shifts of each other. Since  $p$  is prime and  $S$  does not contain any fixed points of this action, each equivalence class must have  $p$  elements. Hence, the cardinality of  $S$  is divisible by  $p$ . □