

## Math 521 – Homework 3

Due Thursday 12, September, 2019 at 10:15am

**Problem 1** (DF 3.1.41). The *commutator* of a group  $G$  is the subgroup of  $G$  generated by the *commutators*  $x^{-1}y^{-1}xy$  commutators of all elements  $x, y \in G$ , denoted

$$[G, G] = \langle x^{-1}y^{-1}xy : x, y \in G \rangle.$$

Prove that  $[G, G]$  is a normal subgroup of  $G$  and that the quotient group  $G/[G, G]$  is abelian.

**Problem 2** (DF 3.3.2). Prove all parts of the Lattice Isomorphism Theorem.

**Problem 3.**

- (a) [DF 3.4.2] Prove part (1) of the Jordan-Hölder Theorem by induction on  $|G|$ .
- (b) Find a composition series for the dihedral group  $D_{30}$ .  
What are the composition factors?