Math 521 – Homework 2

Due Thursday 5, September, 2019 at 10:15am

Problem 1 (DF Exercises 2.1.6 & 2.1.7)**.**

(a) Let G be an abelian group. Prove that $\{g \in G \text{ s.t. } |g| < \infty\}$ is a subgroup of G (called the *torsion subgroup* of G).

(b) Give an explicit example where this set is not a subgroup when G is non-abelian.

(c) Find the torsion subgroup of $\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$.

(d) Show that the set of elements in $\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$ of infinite order together with the identity is *not* a subgroup of this direct product.

Problem 2. Let $G = \mathrm{GL}_2(\mathbb{R})$ and consider the subset of diagonal matrices

$$A = \left\{ \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \text{ s.t. } \lambda_1, \lambda_2 \in \mathbb{R} \backslash \{0\} \right\}.$$

Find $C_G(A)$, $Z(G)$, and $N_G(A)$ (and show that your answers are correct).

Problem 3 (DF Exercise 2.4.14)**.** A group H is called *finitely generated* if there is a finite set $A \subseteq H$ such that $H = \langle A \rangle$.

(a) Prove that every finite group is finitely generated.

(b) Prove that $\mathbb{Z}$ is finitely generated.

(c) Prove that every finitely-generated subgroup of the group $(\mathbb{Q}, +)$ is cyclic.
(If H is a finitely-generated subgroup of $\mathbb{Q}$, show that $H \leq \langle 1/k \rangle$, where k is the product of all the denominators which appear in a set of generators for H.)

(d) Prove that $\mathbb{Q}$ is not finitely generated.