

Math 521 – Homework 1

Due Thursday, August 29, 2019 at 10:15am

Problem 1. Let $p \in \mathbb{Z}_+$ be a prime number.

- (a) Show that the set of non-zero elements of $\mathbb{Z}/p\mathbb{Z}$ form a group under multiplication.
- (b) Show that this does not hold for $p = 6$.

Problem 2. For this exercise, the dihedral group D_{2n} has the usual presentation

$$D_{2n} = \langle r, s \mid r^n = s^2 = 1, rs = sr^{-1} \rangle.$$

- (a) (DF Ch 1.2 Exercise 2) Use the generators and relations above to show that if x is any element of D_{2n} which is not a power of r , then $rx = xr^{-1}$.
- (b) (DF Ch 1.2 Exercise 3) Use the generators and relations above to show that every element of D_{2n} which is not a power of r has order 2. Deduce that D_{2n} is generated by the two elements s and sr , both of which have order 2.

Problem 3 (DF Ch 1.6 Exercise 20). Let G be a group and let $\text{Aut}(G)$ be the set of all isomorphisms from G onto G . Prove that $\text{Aut}(G)$ is a group under function composition. (This is called the *automorphism group* of G and the elements of $\text{Aut}(G)$ are called *automorphisms* of G).