# Math 437 – Homework 9
## Due 10:15am on Thursday, March 30, 2017

Please indicate any sources you used for a given problem on the solution to that problem. For example, if you worked with another student to get the solution to a problem, please indicate who. You are welcome to work together in small groups, but please try the problems on your own first and write up your own solutions.

**Problem 1.** (GS, Example 11.8) The following is a special case of a model, called the Ehrenfest model, that has been used to explain diffusion of gases. We have two urns that, between them, contain four balls. At each step, one of the four balls is chosen at random and moved from the urn that it is in into the other urn. We choose, as states, the number of balls in the first urn $\{0, 1, 2, 3, 4\}$. The transition matrix on states is then

$$
P = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 \\
1/4 & 0 & 3/4 & 0 & 0 \\
0 & 1/2 & 0 & 1/2 & 0 \\
0 & 0 & 3/4 & 0 & 1/4 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}.
$$

(a) Show that this Markov Chain is ergodic but not regular.

(b) Find a stationary distribution.

(c) Suppose we start with zero balls in the first urn and run this Markov chain run for $16,000$ time steps. For each $j = 0, 1, 2, 3, 4$, we count the number of times steps (out of $16,0000$) at which there are exactly $j$ balls in the first urn. What do you expect these counts to be?

**Problem 2.** Alice, Bob, Charlie, and Diane all write online blogs about fun cryptography algorithms. Alice's blog links to the blogs of Bob and Charlie, Bob's blog links to the blogs of Charlie and Diane, Charlie's blog links to the blogs of Diane and Alice, and Diane's blog doesn't link anywhere.

Using the PageRank algorithm with a damping factor of $\alpha = \frac{4}{5}$, find the exact rank of each account, and order the accounts from highest to lowest rank.