

Math 437 – Homework 8

Due 10:15am on Thursday, March 23, 2017

Please indicate any sources you used for a given problem on the solution to that problem. For example, if you worked with another student to get the solution to a problem, please indicate who. You are welcome to work together in small groups, but please try the problems on your own first and write up your own solutions.

Problem 1. Alice is playing the game Rock, Paper, Scissors with Bob. She notices that Bob's behavior can be modeled by a Markov chain. Whenever Bob plays Rock one turn, he plays Paper on the next, and if he plays Paper one turn, he always plays Scissors the next. However, when he plays Scissors, half the time he plays Rock the next turn and half the time he plays Paper.

- Suppose that on the first move, Bob is equally likely to play Rock, Paper, and Scissors. What are the probabilities that Bob will play each of Rock, Paper, and Scissors his second and third turns?
- Is this Markov chain regular?
- After Alice and Bob have been playing for a long, long time what are the proportion of Bob's moves that are Rock, Paper, and Scissors (respectively)?

Problem 2. Consider a Markov chain M on r states with transition matrix P . Let I denote the the $r \times r$ identity matrix. We can form the *lazy version* of M by using the transition matrix $\frac{1}{2}(I + P)$.

- Show that $\frac{1}{2}(I + P)$ a valid transition matrix for a Markov chain.
- Show that the Markov chains given by P and $\frac{1}{2}(I + P)$ have the same steady states.
- Why do you think this is called a lazy version of M ?

Final Project

Your final project should consist of an expository paper roughly 3-5 pages in length related to an application of abstract or linear algebra. You should give a brief description of the topic along with relevant definitions, ideas, and results, but you need not include proofs.

A list of possible final projects topics is available on the course website. You are welcome to pick a topic on the list, subject to my approval. Only one student can work on a given topic and they will be assigned as requested. The timeline is the following:

Thursday, April 6 (or before)	Have approved project topic
Thursday, May 4 at 11am	Final paper due