Math 437 – Homework 12

Please indicate any sources you used for a given problem on the solution to that problem. For example, if you worked with another student to get the solution to a problem, please indicate who. You are welcome to work together in small groups, but please try the problems on your own first and write up your own solutions.

Problem 1. Consider the polynomial ring $\mathbb{Q}[q, d, n, p]$ in variables $q =$ quarters, $d =$ dimes, $n =$ nickels, and $p =$ pennies, and the ideal

$$I = \langle p^5 - n, p^{10} - d, p^{25} - q \rangle \subset \mathbb{Q}[q, d, n, p].$$

(a) Using computational software, compute a Grobner basis for I with respect to $>_{lex}$ and $>_{grlex}$ with $q > d > n > p$.

(b) By hand, compute the normal form of $dn^3 p^7$ with respect to I using $>_{lex}$ and $>_{grlex}$.

(c) Give an interpretation of the normal form of $q^{a_1} d^{a_2} n^{a_3} p^{a_4}$ with respect to $>_{lex}$ and $>_{grlex}$.

Problem 2. (CLO §2.8, #7) Let $S \subseteq \mathbb{R}^3$ be formed by taking the union of straight lines joining pairs of points $(t, 0, 1)$ and $(0, 1, t)$. Then S can be given in the parametric form $(x, y, z) = u(t, 0, 1) + (1 - u)(0, 1, t)$, giving

$$x = ut$$
$$y = 1 - u$$
$$z = u + t - ut$$

(a) Find a polynomial $f(x, y, z)$ for which $V(f)$ contains S.

(b) Show that $V(f) = S$ (that is, every point $(x, y, z) \in V(f)$ is obtained by some values of t, u). Hint: Try to "solve" $f = 0$ for one variable as a function of the other two.