

Math 437 – Homework 11

Due 10:15am on Thursday, April 13, 2017

Please indicate any sources you used for a given problem on the solution to that problem. For example, if you worked with another student to get the solution to a problem, please indicate who. You are welcome to work together in small groups, but please try the problems on your own first and write up your own solutions.

Problem 1. (Graded Reverse Lexicographic Order) For $\alpha \neq \beta \in (\mathbb{Z}_{\geq 0})^n$, say $x^\alpha >_{\text{grevlex}} x^\beta$ if

$$\sum_{i=1}^n \alpha_i > \sum_{i=1}^n \beta_i, \quad \text{or} \quad \sum_{i=1}^n \alpha_i = \sum_{i=1}^n \beta_i \quad \text{and the rightmost nonzero entry of } \alpha - \beta \text{ is negative.}$$

(a) Show that $>_{\text{grevlex}}$ is a monomial ordering. (*Hint:* See Corollary 6 in §2.4 of CLO)

(b) For $f = 5x_1^3 + 4x_2^4 + 3x_1^2x_3^2 + 2x_1 + x_3$, find $LT(f)$ under each of $>_{\text{lex}}$, $>_{\text{grlex}}$, and $>_{\text{grevlex}}$.

Problem 2. (CLO §2.3 #6) Using graded lexicographic order $>_{\text{grlex}}$, find an element g of $\langle f_1, f_2 \rangle = \langle 2xy^2 - x, 3x^2y - y - 1 \rangle \subset \mathbb{Q}[x, y]$ whose remainder upon division by $\{f_1, f_2\}$ is non-zero. Is f_1, f_2 a Gröbner basis for $\langle f_1, f_2 \rangle$? Explain.

Some notes on the final project

Length: Your final project should be roughly 3-5 pages in length, with 1-1.5 line spacing. It's fine if the paper is longer than 5 pages, but not too much longer (10 pages is a hard maximum).

Audience: Think of your audience for this paper as other students in the class, people who have general mathematical background and familiarity with the course material but haven't read in-depth papers on your subject.

Rough draft: If you would like me to read through a rough draft and give you comments, please give me a rough draft by Thursday, April 20. (This is optional!)

Deadline: The paper is due May 4 at 11am. You can submit either a hard copy or pdf.