Math 437 – Homework 10

Due 10:15am on Thursday, April 6, 2017

Please indicate any sources you used for a given problem on the solution to that problem. For example, if you worked with another student to get the solution to a problem, please indicate who. You are welcome to work together in small groups, but please try the problems on your own first and write up your own solutions.

Problem 1. Consider two websites, each linking to each other. Computing the PageRank with dampening factor α involves computing the steady state of the Markov Chain with transition matrix

$$G \;=\; \alpha \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + (1-\alpha) \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \;=\; \begin{pmatrix} (1-\alpha)/2 & (1+\alpha)/2 \\ (1+\alpha)/2 & (1-\alpha)/2 \end{pmatrix}.$$

(a) Compute the eigenvalues and eigenvectors of G (as a functions of α).
(b) Find a formula for G^n.
(c) If $\alpha = .9$, how large should n be so that the first two digits of entries of G^n agree with the limit $\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$. What if $\alpha = .6$?

Problem 2. (GS 11.2, #15) Alice and Bob are playing tennis. Starting from *deuce*, if a player wins the next point, that player has advantage. On the following point, the player with *advantage* either wins the game or the game returns to deuce. Assume that for any point, Alice has probability .6 of winning the point and Bob has probability .4 of winning the point.

(a) Set this up as a Markov chain with the five states $\{s_1 = $ advantage Alice, $s_2 = $ deuce, $s_3 = $ advantage Bob, $s_4 = $ Alice wins, $s_5 = $ Bob wins$\}$.
(b) Starting from deuce, find the expected duration of the game and the probability that Bob will win.

Problem 3. Have an approved topic for your final project.