

Math 407 – Self-Diagnostic Test

Linear algebra and matrix theory are fundamental to the theory and practice of linear optimization. A lack of familiarity or comfort with the material from linear algebra will make this course difficult. There is no time limit for this exam, but if you know the required background material, the test should not take you more than an hour. If you find many of the questions challenging, you should consider carefully before taking this course.

This will not be turned in or graded. Solutions will be posted on the course discussion board. You should try to answer all problems before looking at the solutions.

Problems (1)-(5) below refer to the following system of equations:

$$\begin{aligned}2x_2 + 2x_3 + x_4 + x_5 &= b_1 \\x_1 + x_2 + x_3 + x_4 + 2x_5 &= b_2 \\-x_1 + x_2 + x_3 - x_5 &= b_3\end{aligned}$$

where $\mathbf{b} = (b_1, b_2, b_3)^T \in \mathbb{R}^3$.

- (1) Find a matrix A that expresses the linear system above in the form $A\mathbf{x} = \mathbf{b}$.
- (2) Find a basis for each of the following subspaces:
 - (a) the row span of A
 - (b) the column span of A
 - (c) the right kernel of A , $\{\mathbf{x} : A\mathbf{x} = \mathbf{0}\}$
 - (d) the left kernel of A , $\{\mathbf{y} : \mathbf{y}^T A = \mathbf{0}\}$
- (3) Describe the possible dimensions of the set of solutions to the system of equations $A\mathbf{x} = \mathbf{b}$ (without knowing the value of \mathbf{b}) and give an example \mathbf{b} for each.
- (4) Give a parametrization of $\{\mathbf{x} : A\mathbf{x} = \mathbf{b}\}$ for $\mathbf{b} = (3, 1, 2)^T$.
- (5) Let U be an invertible 3×3 matrix. Describe the row span, column span, right kernel, and left kernel of the matrix UA and their relation to your answers from (2).

Problems (6)-(7) refer to the plane in \mathbb{R}^3 parametrized by

$$\{(5 + t, 1 + s + t, 2s + t) : s, t \in \mathbb{R}\}.$$

- (6) Find the defining equation of this plane.
- (7) Find the point in this plane closest to $(1, 0, 3)$.