Math 407 – Midterm Practice Problems

For Problems (1)-(3), consider the following LP:

$$\max \quad x_1 + 3x_2 \quad \text{such that} \quad x_1 \geq 0, \quad x_2 \geq 0, \quad x_1 + 2x_2 \leq 8, \quad 2x_1 + x_2 \leq 7, \quad x_2 \leq 4$$

(1) Plot the feasible region of the following linear program and solve it graphically.
(2) Reformulate this linear program in equational form.
(3) List an optimal feasible basis of this linear program in equational form and write the corresponding simplex tableau.

For Problems (4), (5), consider the following LP:

$$\max x_1 + x_2 + x_3 \text{ s.t. } \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \geq 0$$

(4) Write down the auxiliary linear program and solve it using the simplex method.
(5) Is the original linear program feasible? Why or why not?

(6) The following simplex tableau occurs while using the simplex method to solve a linear program:

$$\begin{aligned} x_1 &= 3 - x_2 - x_3 \\ x_4 &= x_2 + x_3 - x_5 \\ x_6 &= 1 + x_3 - x_5 \\ z &= x_2 + x_3 - x_5 \end{aligned}$$

Find the optimal value and optimal solution in $\mathbb{R}^6$ of this linear program.

(7) Given a polyhedron $P = \{\mathbf{x} \in \mathbb{R}^n : A\mathbf{x} \leq \mathbf{b}\}$ where $A \in \mathbb{R}^{m \times n}$ and $\mathbf{b} \in \mathbb{R}^m$ with a vertex $\mathbf{v} \in P$, show that the set

$$\{\mathbf{c} \in \mathbb{R}^n : \text{ the maximum of } \mathbf{c}^T\mathbf{x} \text{ over } P \text{ is attained at } \mathbf{v}\}$$

is convex.