Math 308 P Conceptual Problems #4
Due Wednesday, February 6

Please write your name and your quiz section (PA, PB, or PC) on your homework paper.

(1) (Geometry Question) Suppose we are given the unit square $\mathcal{S}$ in $\mathbb{R}^2$ with corners $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.

(a) Find a linear transformation T that sends $\mathcal{S}$ to the parallelogram $\mathcal{P}$ with corners $(0,0)$, $(1,2)$, $(2,2)$ and $(1,0)$.
(b) Where does T send the point $(1/2, 1/2)$? (Note this is the center point of $\mathcal{S}$.)
(c) Is the linear transformation T unique? Why or why not?
(d) Find all linear transformations T' which send $\mathcal{S}$ to itself.
(e) Suppose we want to send $\mathcal{S}$ to the parallogram $\mathcal{Q}$ obtained by moving $\mathcal{P}$ by one unit in the horizontal direction. What transformation can do this? (The answer will not be a linear transformation, but still you should be able to find a formula for it.)
(f) How can you map $\mathcal{S}$ to a parallelogram $\mathcal{R}$ of area 4 which has $(0,0)$ and $(1,0)$ as two of its corners?
(g) What is the general formula for the linear transformation that sends $\mathcal{S}$ to a parallelogram of area k while still keeping $(0,0)$ and $(1,0)$ as two of its corners?

(2) Let
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \end{bmatrix}.$$
Find a 3×2 matrix B with $AB = I_2$. Is there more than one matrix B with this property? Justify your answer.

(3) Find a 3×2 matrix A and a 2×3 matrix B such that AB is invertible or explain why such matrices cannot exist. Answer the same question with the requirement that BA be invertible.