Math 308 P Conceptual Problems #2

Due Wednesday, January 16

(1) Matt is a software engineer writing a script involving 6 tasks. Each must be done one after the other. Let t_i be the time it takes to do the ith task. These times have a certain structure:

- The time it takes to do the first three tasks is half as long as the time it takes to do the next two tasks.
- The time it takes to do the last two tasks is twice as long as the time it takes to do the previous three tasks.
- The second task takes 1 second.
- The fourth task takes 10 seconds.

(a) Write an augmented matrix for the system of equations describing the length of each task.

(b) Reduce this augmented matrix to reduced echelon form.

(c) Suppose he knows additionally that the sixth task takes 20 seconds and the first three tasks will run in 50 seconds. Write the extra rows that you would add to your answer in (b) to take account of this new information.

(d) Solve the system of equations in (c) to find out how long it takes to do each task.

(2) Before paying employee bonuses and state and federal taxes, a company earns profits of \$103,000. The company pays employees a bonus equal to 5% of after-tax profits. State tax is 5% of profits (after bonuses are paid). Finally, federal tax is 40% of profits (after bonuses and state tax are paid). Calculate the amounts paid in bonuses, state tax and federal tax.

(3) (a) Use Gauss-Jordan elimination to find the general solution for the following system of linear equations:

$$\begin{aligned} z_2 + 3z_3 - z_4 &= 0 \\ -z_1 - z_2 - z_3 + z_4 &= 0 \\ -2z_1 - 4z_2 + 4z_3 - 2z_4 &= 0 \end{aligned}$$

(b) Give an example of a solution to the previous system of linear equations.

(c) The points $(1, 0, 3)$, $(1, 1, 1)$, and $(-2, -1, 2)$ lie on a unique plane $a_1x_1 + a_2x_2 + a_3x_3 = b$. Using your previous answers, find an equation for this plane. (Hint: think about the relationship between the previous system and the one you would need to solve in this question.)