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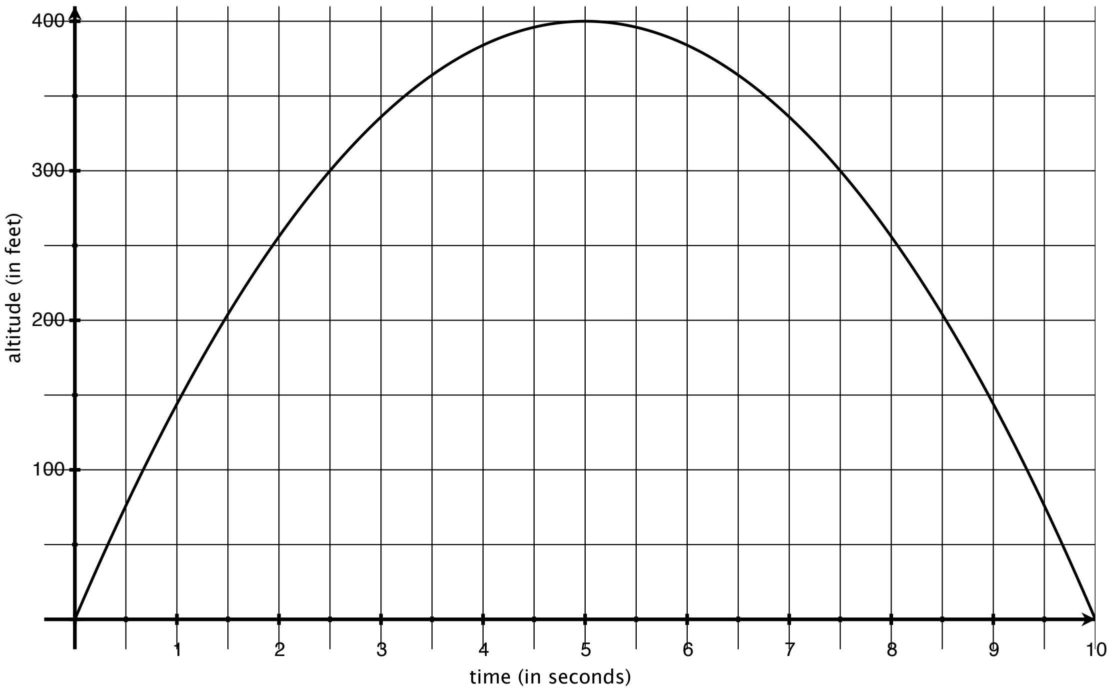
Math 112

Group Activity: The Vertical Speed of a Shell

A shell is fired straight up by a mortar. The graph below shows its altitude as a function of time.



The function describing the shell's altitude at time t will be denoted by $f(t)$, where altitude is in feet and t is in seconds.

1. Use the altitude graph to approximate the following quantities:

   (a) the time required for the shell to reach the altitude 300 ft for the first time

   (b) the change in altitude from time $t = 2$ to $t = 4$

   (c) the average speed of the shell from time $t = 1$ to time $t = 4$

The formula for the altitude (in feet) after t seconds is given by: $f(t) = 160t - 16t^2$.

2. Use this formula to compute the exact values of the quantities from question #1. Check these values against the approximations you read from the graph.

   (a) the time required for the shell to reach the altitude 300 ft for the first time

   (b) the change in altitude from time $t = 2$ to $t = 4$

   (c) the average speed of the shell from time $t = 1$ to time $t = 4$

Imagine that there is a tiny speedometer in the shell that records the shell's velocity just as the speedometer in your car records the car's velocity. This speedometer measures the shell's *instantaneous speed*. This is the speed the shell is traveling, not over an interval of time, but at one specific instant. We usually think of speed as distance traveled divided by time elapsed, but over a single instant, distance traveled and time elapsed are both 0 and speed is not so simple. Finding instantaneous speeds is what calculus is all about. We'll start by making some approximations.

3. We'll approximate the shell's instantaneous speed at $t = 2$ by finding the shell's average speed over a tiny time interval (an interval of length 0.01 seconds). We'll do this in two different ways:

   (a) **Graphically:** On the graph of altitude, draw the line whose slope gives the shell's average speed from $t = 2$ to $t = 2.01$. That is, draw the line that goes through the graph of altitude at $t = 2$ and $t = 2.01$. Note that, on this scale, it's difficult to distinguish between $t = 2$ and $t = 2.01$. Do your best to draw such a line, extending it out as far as you can in both directions. Then estimate the coordinates of two points on the line and compute its slope. (Regardless of your choice of points, this gives the desired average speed. Make sure you understand why.)

   (b) **Algebraically:** Use the formula for altitude to compute the shell's average speed from $t = 2$ to $t = 2.01$. That is, compute

   $$\frac{f(2.01) - f(2)}{0.01}.$$

   Compare with your answer from part (a). Would you agree that this is a good approximation of the shell's speed at the instant $t = 2$?

4. Use the same two methods to approximate the shell's instantaneous speed at $t = 5$, when it reaches its peak height. Instantaneous speed at $t = 5$ will be approximately equal to the average speed from $t = 5$ to $t = 5.01$.

   (a) **Graphically:**

   (b) **Algebraically:**

5. Use the same two methods to approximate the shell's instantaneous speed at $t = 0$, the instant it was launched. Instantaneous speed at $t = 0$ will be approximately equal to the average speed from $t = 0$ to $t = 0.01$.

   (a) **Graphically:**

   (b) **Algebraically:**

6. In #3, you approximated the shell's speed at $t = 2$ by computing the average speed over the interval from $t = 2$ to $t = 2.01$. This is a decent approximation of the speed at the instant $t = 2$ but we can get even better approximations by using even shorter time intervals.

   (a) Use the algebraic method to compute the average speed of the shell over the interval from $t = 2$ to $t = 2.001$.

   (b) Use the algebraic method to compute the average speed of the shell over the interval from $t = 2$ to $t = 2.0001$.

   (c) What would you guess is the exact value of the instantaneous speed of the shell at $t = 2$?

   (d) Describe how you could interpret the instantaneous speed of the shell at $t = 2$ on the graph of altitude as the slope of a line.

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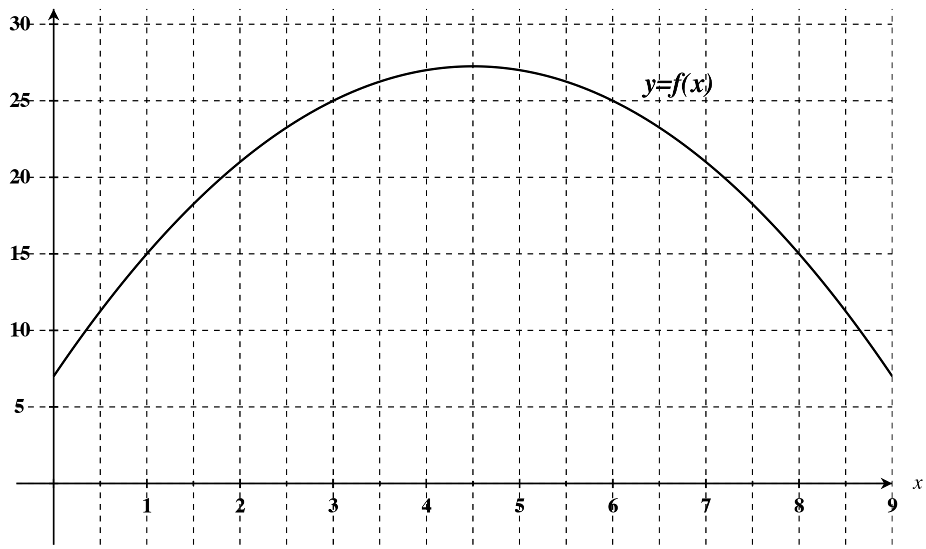
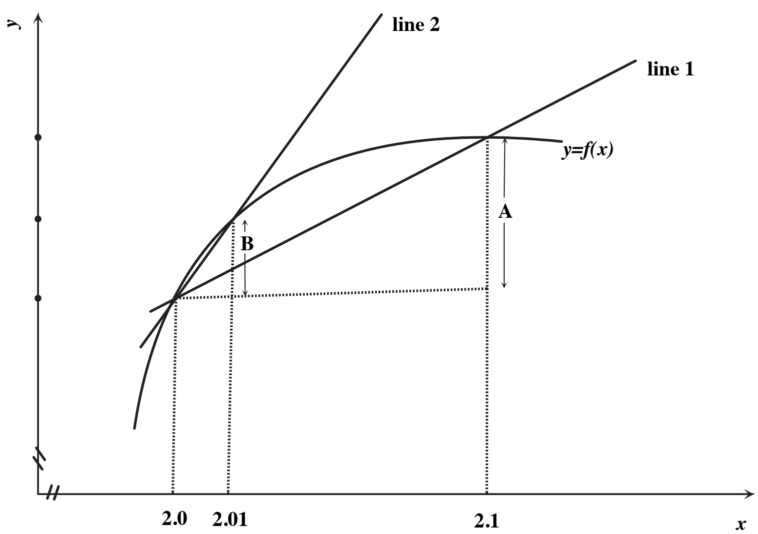
Math 112
Group Activity: Tangents and Secants

The graph below is of a function $y = f(x)$.



1. Draw a tangent line to the graph of $f(x)$ at $x = 2$ and compute its slope. What is the functional notation for the slope you just computed?

2. Below is a magnification of the graph of $f(x)$ near $x = 2$. (It's not to scale and I've exaggerated the curviness of $f(x)$.) There are three points marked on the y-axis. Their heights are $f(2.0)$, $f(2.1)$ and $f(2.01)$. Label them correctly.



3. In terms of $f(2.0)$, $f(2.1)$ and $f(2.01)$, what are the values of the lengths marked A and B and the slopes of line 1 and line 2 in the picture above?

$A =$ slope of line 1 $=$

$B =$ slope of line 2 $=$

4. Which is closer to the slope of the tangent line to $f(x)$ at $x = 2$: the slope of line 1 or the slope of line 2?

5. How might you draw a line whose slope is even closer to the slope of the tangent to $f(x)$ at $x = 2$?

6. The formula for $f(x)$ is: $f(x) = -x^2 + 9x + 7$.

   (a) The slope of line 1 is given by $\frac{f(2.1) - f(2.0)}{0.1}$. Use the formula for $f(x)$ to compute the exact value of this slope.

   (b) Use the formula for $f(x)$ to compute the exact value of the slope of line 2.

   (c) Your work in parts (a) and (b) forms the first two lines of the following table. We've completed the next two lines for you. Use the pattern established in the table to complete the last two lines of the table. (Do not try to use your calculator to finish the table—just follow the pattern that you see emerging. Most calculators will round and the rounding error will cause the pattern stop.)

h	$2+h$	$f(2+h)$	f(2)	$f(2+h) - f(2)$	$\frac{f(2+h) - f(2)}{h}$
0.1	2.1	21.49	21	0.49	4.9
0.01	2.01	21.0499	21	0.0499	4.99
0.001	2.001	21.004999	21	0.004999	4.999
0.0001	2.0001	21.00049999	21	0.00049999	4.9999
0.00001					
0.000001					

   (d) Do you agree that the right-most column of the table gives slopes of secant lines that are getting closer and closer to the tangent line to $f(x)$ at $x = 2$? What would you predict is the exact value of $f'(2)$, the slope of the tangent line to $f(x)$ at $x = 2$? Is this close to the slope you computed using the graph in #1?

7. We'll use a similar method to compute $f'(3)$, the slope of the line tangent to $f(x)$ at $x = 3$, but this time we'll use algebra to make a smaller table. Again, the formula for $f(x)$ is:

$$f(x) = -x^2 + 9x + 7.$$

(a) Compute and simplify the expression $\dfrac{f(3+h) - f(3)}{h}$.

(b) What is the graphical interpretation of the expression in part (a)? (Your answer should begin "It is the slope of the...".)

(c) Use your answer to part (a) to quickly fill in the following table:

h	$\dfrac{f(3+h) - f(3)}{h}$
0.1	
0.01	
0.001	
0.0001	

<— evaluate this using your answer to (a)

(d) Do you agree that the right-most column of the table gives slopes of secant lines that are getting closer and closer to the tangent line to $f(x)$ at $x = 3$? What would you predict is the exact value of $f'(3)$, the slope of the tangent line to $f(x)$ at $x = 3$?

(e) On the graph at the beginning of this activity, draw the line tangent to $f(x)$ at $x = 3$ and compute its slope. Is it close to the value you just found for $f'(3)$?

8. Finally, let's find a function that gives $f'(a)$, the slope of the tangent line to $f(x)$ at $x = a$. Again,

$$f(x) = -x^2 + 9x + 7.$$

(a) Compute and simplify the expression $\dfrac{f(a+h) - f(a)}{h}$.

(b) What is the graphical interpretation of the expression in part (a)? (Your answer should begin "It is the slope of the...".)

(c) If you take progressively smaller values for h, your answer to part (a) approaches some value that depends on a. What is that value, what is its graphical interpretation, and how would you use functional notation to express it?

(d) Use your answer to part (c) to compute $f'(2)$ and $f'(3)$. Do these match your previous computations?

(e) Use your answer to part (c) to compute $f'(1)$ and $f'(5)$.

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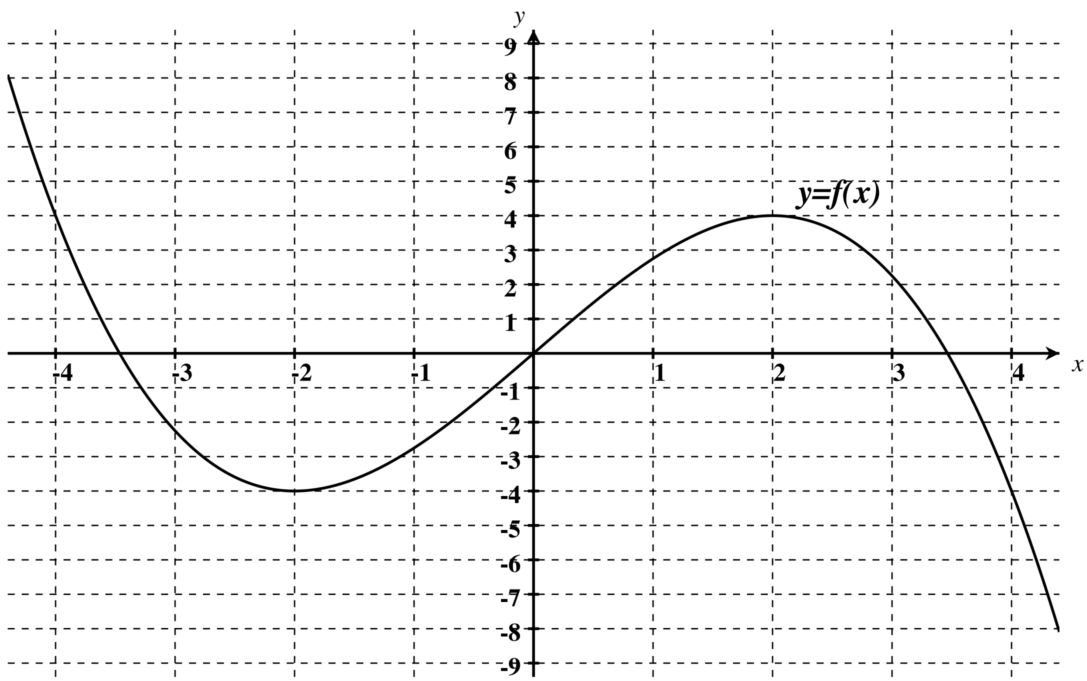
Math 112

Group Activity: Graphs of Derivatives

The graph below shows the function $y = f(x)$.



1. Recall that $f'(a)$ gives the slope of the line tangent to $f(x)$ at $x = a$.

   (a) List all values of a at which $f'(a) = 0$.

   (b) Give four values of a at which $f'(a)$ is *negative*.

   (c) Give three values of a at which $f'(a)$ is *positive*.

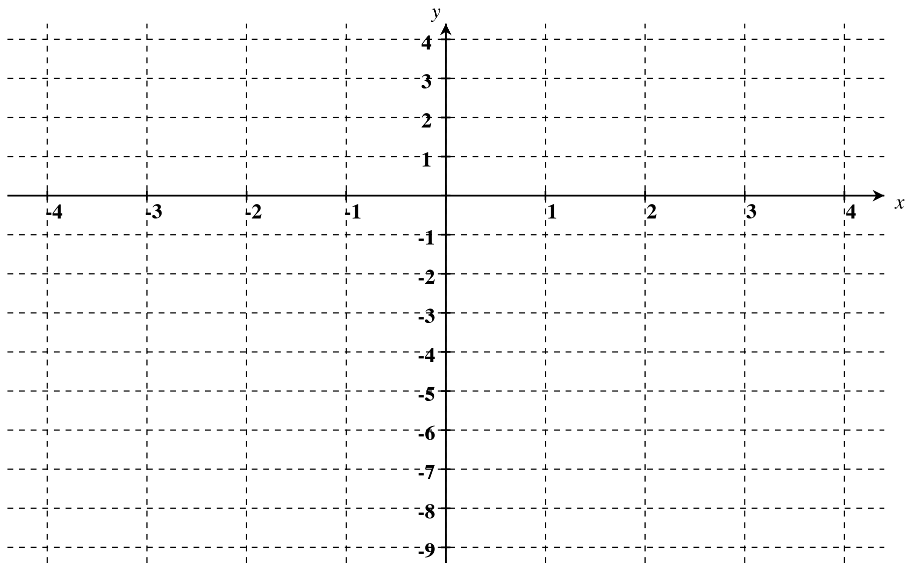
   (d) Describe how you can tell by looking at the graph of $f(x)$ whether the value of $f'(a)$ will be positive, negative, or zero.

   (e) This graph has some nice symmetry. Without computing any slopes, use your ruler to verify that the following are true for this graph: $f'(-1) = f'(1)$, $f'(-3) = f'(3)$, and $f'(-4) = f'(4)$.

   (f) The following table contains some of the values of $f'(x)$. Fill in the remaining entries in the table. To fill in each entry, either use entries already in the table along with the symmetry described in part (e) or draw an appropriate tangent line and compute its slope.

x	-4	-3	-2	-1	0	1	2	3	4
$f'(x)$	-9			2.25				-3.75	

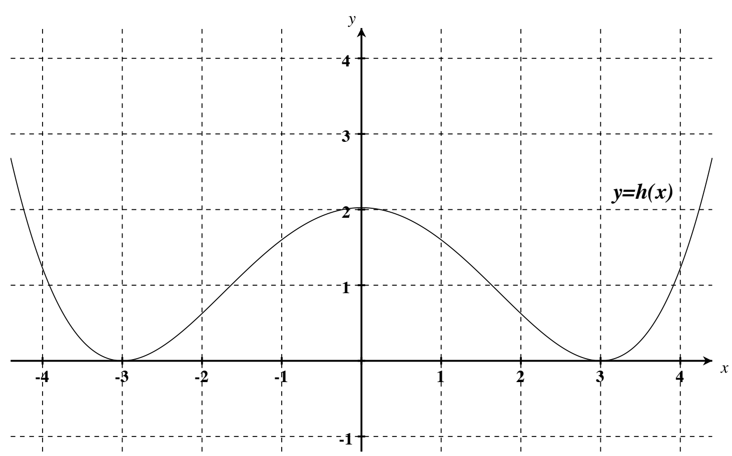
(g) Use the values in the table to sketch the graph of $f'(x)$ on the axes below. Notice that slopes of tangent lines to the graph of $f(x)$ become y-values on the graph of $f'(x)$.



Notice that:

- The graph of $f'(x)$ crosses the x-axis at those values of x where $f(x)$ has horizontal tangent lines.
- The graph of $f'(x)$ is *below* the x-axis (i.e., its y-values are *negative*) on the intervals where the graph of $f(x)$ is *decreasing*.
- The graph of $f'(x)$ is *above* the x-axis (i.e., its y-values are *positive*) on the intervals where the graph of $f(x)$ is *increasing*.

2. Now consider the following graph of a function $h(x)$.

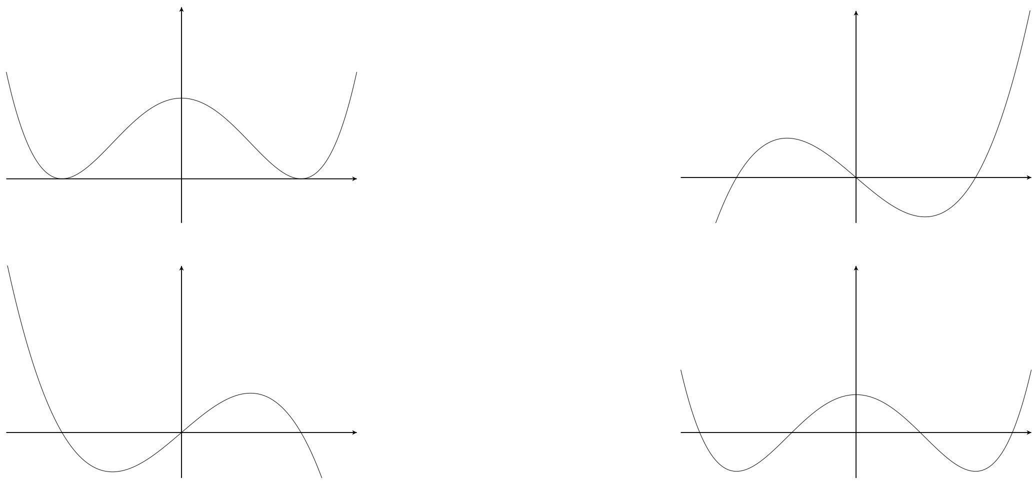


(a) List all values of x at which the graph of $h'(x)$ crosses the x-axis.

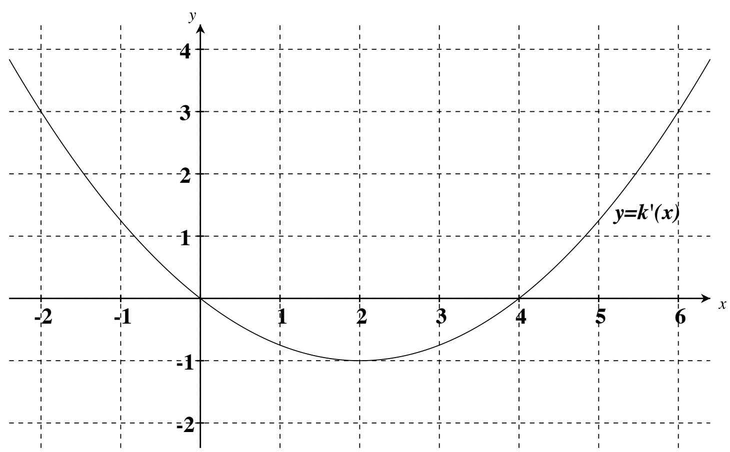
(b) List all intervals on which the graph of $h'(x)$ is *below* the x-axis.

(c) List all intervals on which the graph of $h'(x)$ is *above* the x-axis.

(d) Which of the following is the graph of $h'(x)$?



3. Below is the graph of $k'(x)$, the derivative of a function $k(x)$. The graph of $k(x)$ is not shown.

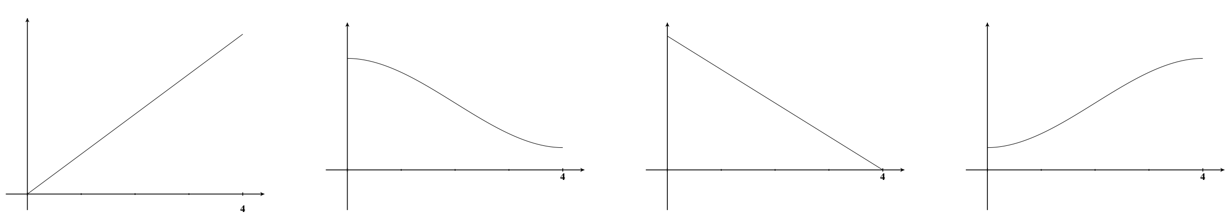


(a) List all values of x at which the graph of $k(x)$ has horizontal tangent lines.

(b) List all intervals on which the graph of $k(x)$ is *increasing*.

(c) List all intervals on which the graph of $k(x)$ is *decreasing*.

(d) Which of the following could be the graph of $k(x)$ on the interval from $x = 0$ to $x = 4$?



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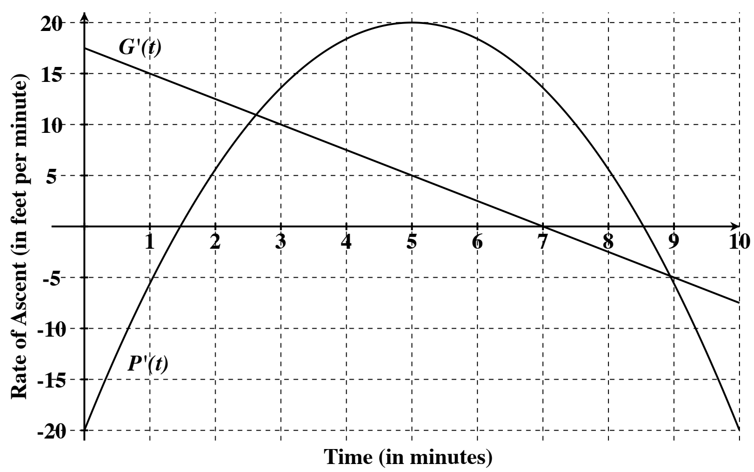
Math 112

Group Activity: Rate of Ascent Graphs

A Purple balloon and a Green balloon rise and fall. When we start watching, at $t = 0$, both balloons are 30 feet above the ground. Their altitudes at time t minutes are given by functions $P(t)$ and $G(t)$, both measured in feet. The graphs below show the *instantaneous rates of ascent* of the balloons. That is, the graphs show $P'(t)$ and $G'(t)$. So, for example, at $t = 1$, the rate of ascent of the Green balloon is 15 feet per minute, which means that the Green balloon's altitude is *increasing* (i.e., the Green balloon is rising) at a rate of 15 feet per minute. Similarly, at $t = 1$, the rate of ascent of the Purple balloon is -5 feet per minute, which means that the Purple balloon's altitude is *decreasing* (i.e., the Purple balloon is falling) at a rate of 5 feet per minute.



1. At $t = 0$, the rate graph shows that the Green balloon's rate of ascent is 17.5 and the Purple balloon's rate is -20. What does that mean about the balloons when we first start observing? Are they rising or falling?

2. During the first 1.5 minutes, the Green balloon's rate graph decreases from 17.5 to 13.75. What does that mean about the Green balloon? Is it rising or falling? Is it getting faster or slower?

3. The Purple balloon's rate graph is negative during the first 1.5 minutes. What does that mean about the Purple balloon? How does the Purple balloon's rate change during the first 1.5 minutes? Is the balloon getting faster or slower?

4. During the first 1.5 minutes are the balloons getting farther apart or closer together? (Think about whether the balloons are rising or falling during this time period.) Which balloon is higher at $t = 1.5$?

5. You can't tell their exact altitudes by looking at the rate graphs, but suppose that you are told that at $t = 1.5$ minutes, the Green balloon is 54 feet higher than the Purple balloon. Over the next 1.1 minutes, both balloons are rising (how can you tell?) and Green's rate graph is above Purple's rate graph. Are the balloons getting farther apart or closer together from $t = 1.5$ to $t = 2.6$ minutes?

6. The two rate graphs cross at 2.6 minutes. This means that the balloons are rising at the same **rate** at that instant. But they do not have the same **altitude** at $t = 2.6$. In fact, since the Green balloon has always been rising faster than the Purple balloon, the two balloons are farther apart at $t = 2.6$ than at any previous time, with the Green balloon much higher than the Purple balloon.

   Over the next few minutes, beginning at $t = 2.6$, both balloons are rising but the Purple balloon's rate graph is above the Green balloon's rate graph. Are the balloons getting farther apart or closer together during the next few minutes beginning at $t = 2.6$? Explain.

7. Do you agree with the following statement?

   > *During the first four minutes, the balloons are farthest apart at $t = 2.6$, when their rate graphs are crossing.*

   Describe an analogous situation in the context of revenue, cost, and profit.

8. The Green balloon's rate is positive until $t = 7$ and then it becomes negative. Describe what you would see the Green balloon do at $t = 7$.

9. On the interval from $t = 7$ to $t = 8$, determine whether each balloon is rising or falling and whether each balloon is getting faster or slower.

10. Again, you can't tell by looking at the rate graphs, but suppose you're told that at $t = 8.5$ minutes, the Purple balloon is approximately 32 feet higher than the Green balloon. Describe what happens to the balloons from $t = 8.5$ to $t = 10$. Are they rising or falling? Getting faster or slower? (BONUS: Do they get closer together or farther apart?)

11. On a separate sheet of paper, draw the basic shapes of the altitude graphs for each balloon, each on a separate set of axes. Don't worry about getting the exact altitudes, just note where each graph is increasing and decreasing and the times at which the balloons change direction (from rising to falling or from falling to rising).

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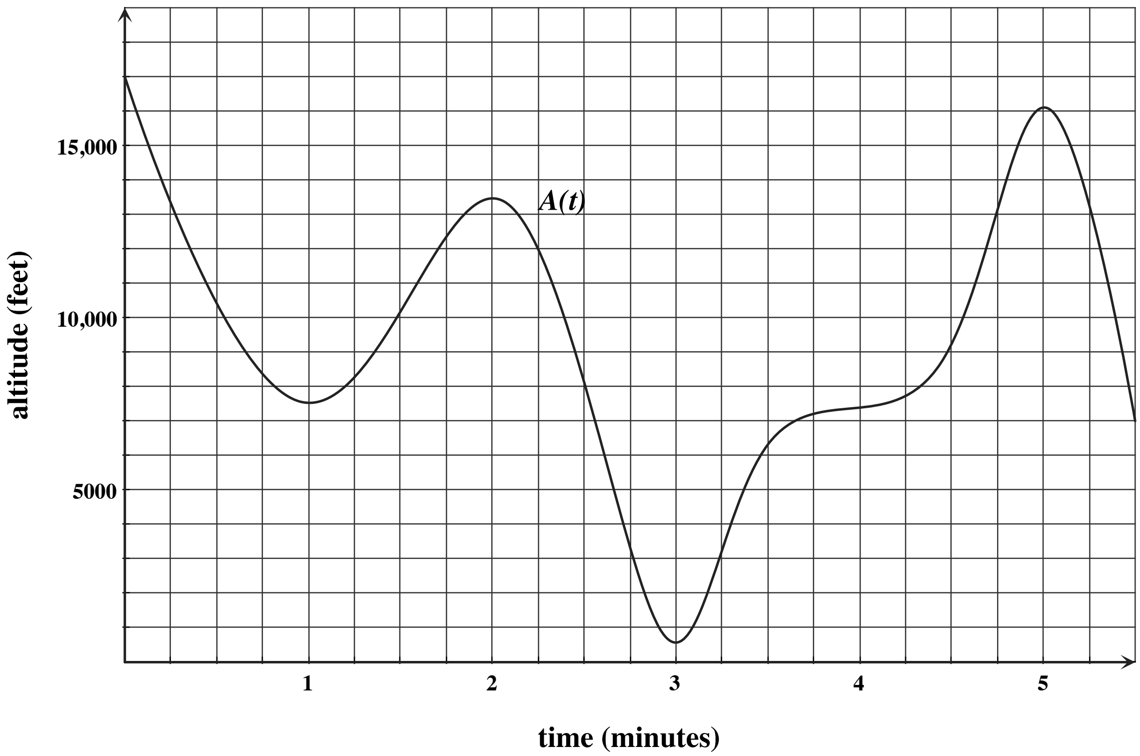
Math 112

Group Activity: Local vs. Global Optima

1. Below is the graph of altitude, $A(t)$, for a balloon that is rising and falling.



(a) List all critical values of the function $A(t)$.

(b) Which of these give local (relative) maxima? local minima? horizontal points of inflection?

(c) On the interval from $t = 0$ to $t = 5.5$, the balloon is at its highest altitude at $t = 0$. We say that the **global maximum** of $A(t)$ *occurs at* $t = 0$. What is that altitude?

(d) At what time does the **global minimum** of $A(t)$ (the *lowest* altitude) occur? What is the lowest altitude the balloon reaches on the interval shown?

(e) We're now going to restrict our attention to smaller time intervals. Fill in the following chart with the values of t_{max}, the value of t that gives the global maximum altitude on the given interval, and t_{min}, the value of t that gives the global minimum altitude on the given interval.

Interval	t_{max}	t_{min}
from $t = 0$ to $t = 5.5$	0	3
from $t = 0.75$ to $t = 2.25$		
from $t = 1.75$ to $t = 2.75$		
from $t = 4.5$ to $t = 4.75$		

The previous exercise demonstrates the following fact: On the interval from $x = a$ to $x = b$, the global maximum value of a function $f(x)$ occurs either at a critical value or at one of the endpoints of the interval. That is, the global maximum value of $f(x)$ is either a local maximum value of $f(x)$ OR it is $f(a)$ or $f(b)$. Similarly, the global minimum value of $f(x)$ is either a local minimum value of $f(x)$ OR it is $f(a)$ or $f(b)$.

This gives a convenient process to find the local and global maximum and minimum vlaues of a function $f(x)$ on the interval from $x = a$ to $x = b$.

Step 1: Compute $f'(x)$, set $f'(x) = 0$, and solve for x. This gives the critical values for $f(x)$.

Step 2: Evaluate $f(x)$ at each of the critical values **that lie between** a **and** b (you can ignore any critical values that are not in your interval). Also, compute $f(a)$ and $f(b)$.

Step 3: Use the information gathered in Steps 1 and 2 to SKETCH A ROUGH GRAPH of $f(x)$ on the interval from a to b. You should be able to see on your graph all local and global optima.

2. The total revenue and total cost (both in hundreds of dollars) for selling q hundred Shrubnods are given by:

$$TR(q) = -0.08q^2 + 2.35q \text{ and } TC(q) = 0.01q^3 - 0.3q^2 + 3q + 4.$$

(a) Compute the formula for profit $P(q)$ and its derivative $P'(q)$.

(b) Find all critical values of the profit function. (You may round your answers to 2 digits after the decimal.)

(c) If you produce between 10 hundred and 15 hundred Shrubnods, what production level will yield the largest profit? (Follow the three-step method given above: You've already done Step 1. Evaluate $P(q)$ at any relevant critical values and the endpoints of your interval. Use that information to sketch the graph of $P(q)$ from $q = 10$ to $q = 15$. Then you can see the optima.)

(d) If you produce between 9 hundred and 10 hundred Shrubnods, what production level will yield the largest profit?

(e) If you can produce any number of Shrubnods, what production level will yield the largest profit? What is the largest possible profit for producing Shrubnods? (To answer this question, sketch a rough graph of the entire profit function. Include the correct "y"-intercept and all critical points.)

(f) What is the global minimum value of profit on the interval from $q = 0$ to $q = 5$ hundred Shrubnods? (NOTE: Your answer should be negative. If the smallest possible profit is negative, then its absolute value is the largest possible loss.)

(g) If you can produce any number of Shrubnods, what production level will cause you to lose the most money? What is the largest possible loss for producing Shrubnods?

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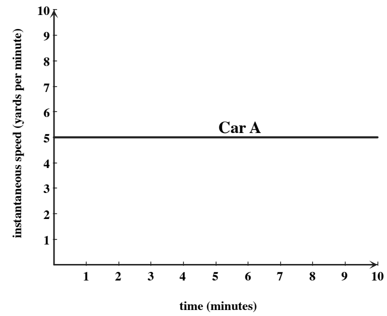
Math 112

Group Activity: Distance Traveled from Speed

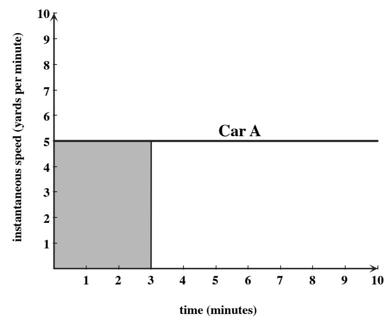
Recall: Instantaneous speed is the derivative of distance traveled. Given a formula for distance traveled, $D(t)$, we simply compute the derivative $D'(t)$ to obtain instantaneous speed at time t. Given the graph of distance traveled, to compute instantaneous speed at time t, we find the slope of the line tangent to distance traveled at that time. The slope of a tangent line to the graph of distance traveled becomes a "y"-value on the graph of instantaneous speed.

Goal: To investigate how we find values of distance traveled given the graph of instantaneous speed.

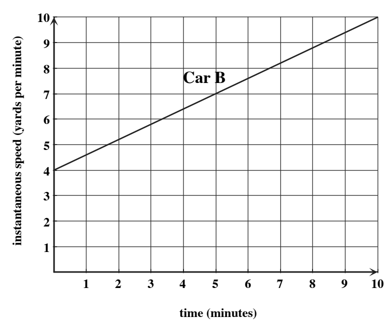
1. The graph at right shows the instantaneous speed of Car A. Car A travels at a constant speed of 5 yards per minute. Compute the distance Car A travels during the first three minutes. (How far does it go if it travels at a constant speed of 5 yards per minute for 3 minutes?)



2. Compute the *area* under Car A's speed graph and above the t-axis from $t = 0$ to $t = 3$. What do you notice?



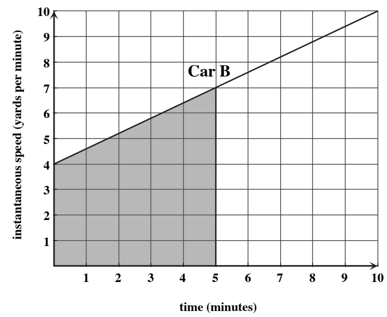
3. The graph at right shows the instantaneous speed of Car B. Car B does not travel at a constant speed—it is getting faster all the time. How fast is Car B traveling at $t = 0$? At $t = 5$?



4. The following is always true: if an object travels with a linear speed, its average speed during that time interval is exactly half-way between its highest and lowest speeds on that interval. Use this fact and your answer to the previous question to determine the *average speed* of Car B on the interval from $t = 0$ to $t = 5$.

5. We know that average speed $= \dfrac{\text{distance traveled}}{\text{time elapsed}}$. Use this fact and your answer to the previous question to compute the distance Car B travels from $t = 0$ to $t = 5$.

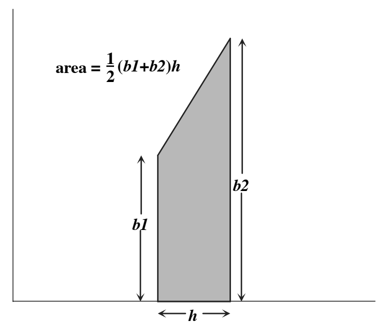
6. Compute the *area* under Car B's speed graph and above the t-axis from $t = 0$ to $t = 5$. (It may help to view this region as a triangle on top of a rectangle.) What do you notice?



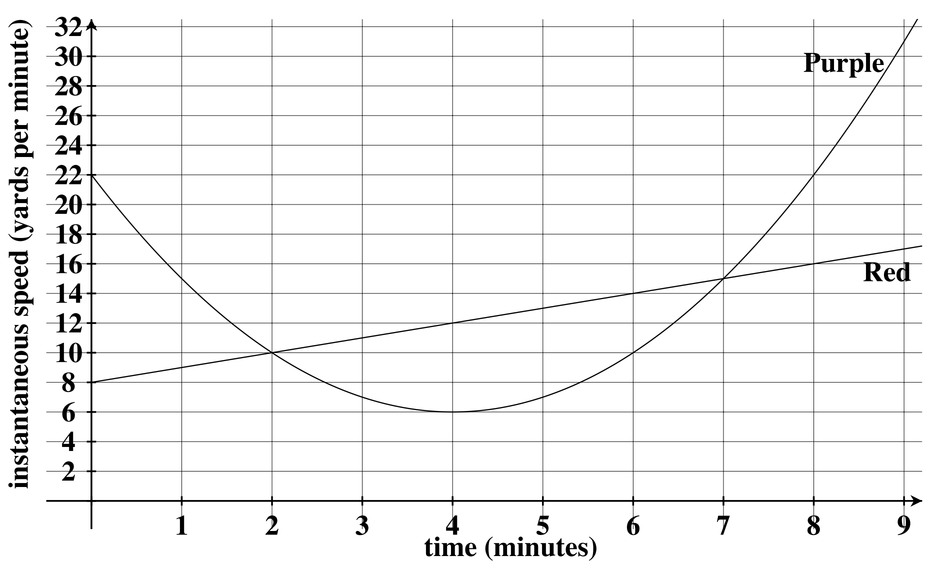
In general, given the graph of instantaneous speed, distance traveled from $t = a$ to $t = b$ is equal to the *area* of the region under the speed graph and above the t-axis from $t = a$ to $t = b$.

7. Use the graph above to compute the distance Car B travels from $t = 5$ to $t = 10$. (Again, view the region whose area you're computing as a triangle on top of a rectangle.)

For the rest of this activity, you'll be computing a lot of areas. Most of the regions whose areas you'll need will be trapezoids. The graph at right shows a quick formula for computing the area of a trapezoid of the type you will see.



8. A red car and a purple car travel along a straight track. At $t = 0$, both cars are next to each other. The graph below shows their instantaneous speeds. We'll use areas to find the graph of distance traveled for each car.



(a) On the interval from $t = 0$ to $t = 1$, the graph of the Red car's speed is linear. Compute the area under the Red car's speed graph on this interval to find the distance Red travels from $t = 0$ to $t = 1$.

(b) The Purple car's speed graph is *not* linear. However, on a short enough interval, a curvy graph looks *almost* linear. In particular, on the interval from $t = 0$ to $t = 1$, the area under the Purple car's speed graph can be approximated by the area of a trapezoid. Compute this area to estimate the distance Purple travels in the first minute.

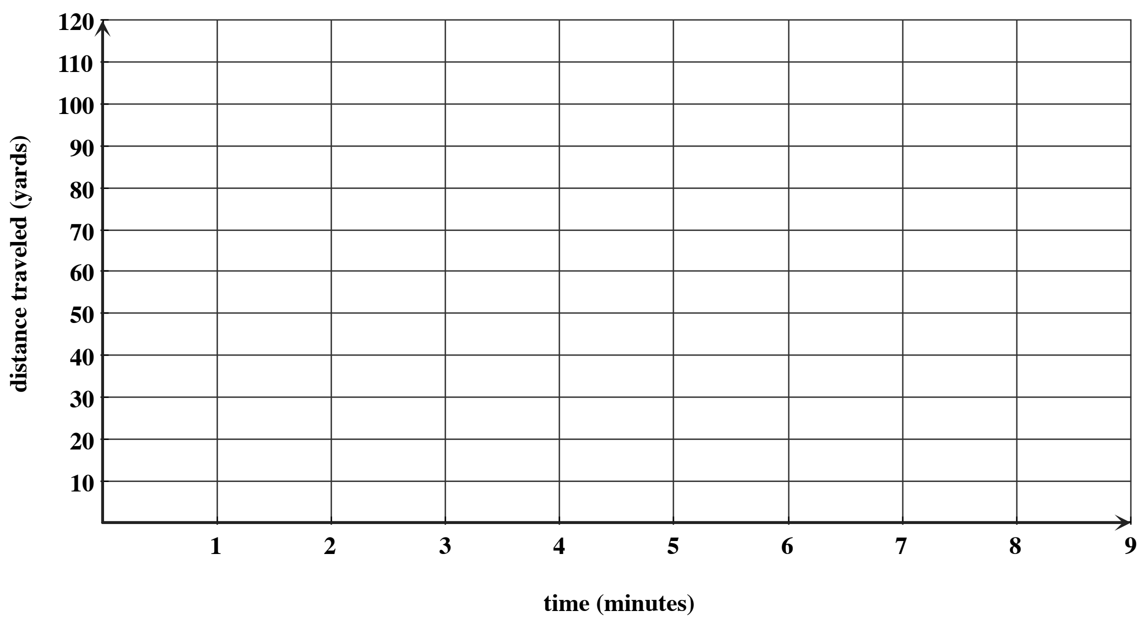
(c) Check your answers to parts (a) and (b) against the first entry for each car in the following table and compute the appropriate areas to fill in the remaining entries.

Interval	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–8	8–9
Distance covered by Red car	8.5	9.5	10.5			13.5		15.5	16.5
Distance covered by Purple car	18.5	12.5	8.5		6.5				26.5

(d) The table in the previous question gives us distance traveled by each car over one-minute intervals. To get distance traveled over wider intervals, we can add these areas together. For example, in the interval from $t = 0$ to $t = 3$, the Purple car travels approximately $18.5 + 12.5 + 8.5 = 39.5$ yards. Add the appropriate areas to fill in the following table, giving distance traveled from time 0 to time t.

time t	0	1	2	3	4	5	6	7	8	9
Distance covered by Red car from 0 to t	0	8.5	18	28.5						
Distance covered by Purple car from 0 to t	0	18.5	31	39.5						

(e) Sketch the graphs of distance traveled for the Red car and the Purple car on the axes below.



(f) Use the distance graphs to estimate the times at which the two cars are farthest apart. What's true about the speed graphs at those times?

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Math 112

Group Activity: Total Revenue and Total Cost from Marginal Revenue and Marginal Cost

<u>Recall:</u>

- The graphs of total revenue and variable cost go through the origin:
  - $TR(0) = 0$
  - $VC(0) = 0$
- The "y"-intercept of total cost is fixed cost: $TC(0) = FC$.
- Total cost is the sum of variable cost and fixed cost, which means that the graph of total cost is a vertical shift of the graph of variable cost.
$$TC(q) = VC(q) + FC.$$
- The derivative of total revenue is marginal revenue: $MR(q) = TR'(q)$.
- The derivative of total cost and the derivative of variable cost are the same. Both are equal to marginal cost:
$$TC'(q) = VC'(q) = MC(q).$$

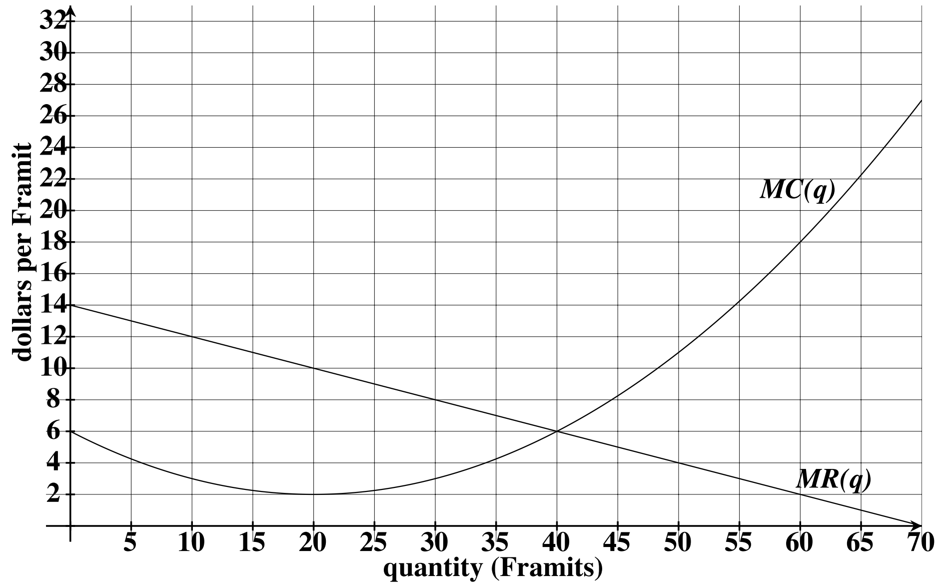
1. (a) Find $MR(q)$ if $TR(q) = -\frac{3}{2}q^2 + 20q$.

   (b) Find $TR(q)$ if $MR(q) = 100 - 9q$.

   (c) Find FC, $VC(q)$, and $MC(q)$ if $TC(q) = \sqrt{q + 64}$.

   (d) Find $VC(q)$ and $TC(q)$ if $MC(q) = 30\sqrt{q + 100}$ and $FC = 50,000$.

2. The graph below shows the graphs of marginal revenue and marginal cost to sell and produce Framits.



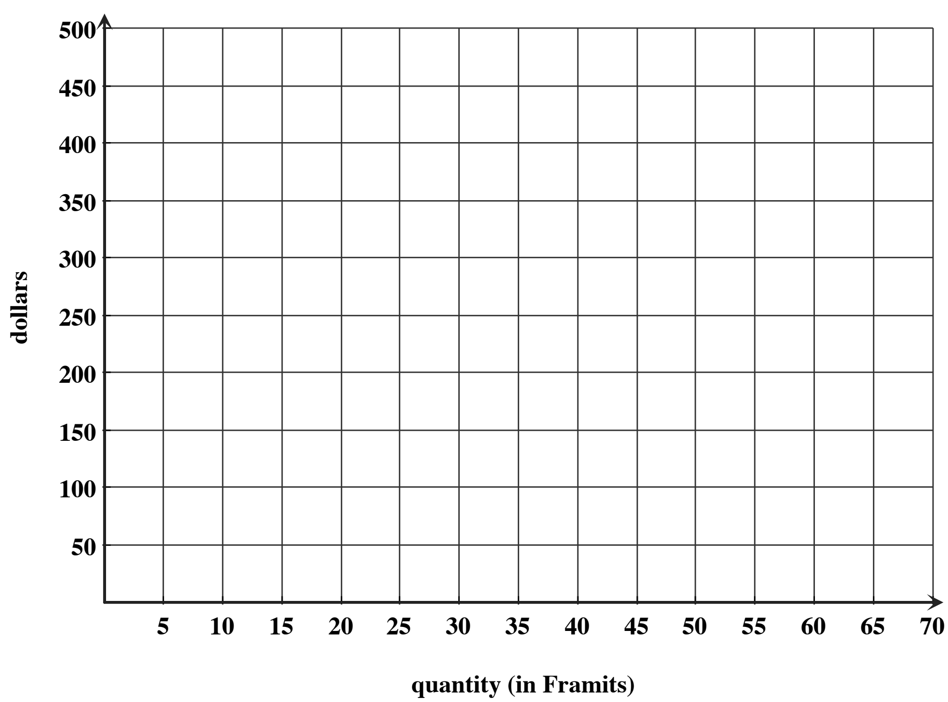
(a) Define a function

$$A(q) = \text{the area under } MR \text{ from 0 to } q.$$

Fill in the values in the following table:

q	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70
$A(q)$	0	67.5	130			287.5			400		450				490

(b) Sketch the graph of $A(q)$ on the following set of axes:



We've learned in previous activities that such an area function gives us an anti-derivative of $MR(q)$. Moreover, this is the anti-derivative of $MR(q)$ that goes through the origin. Thus, you have just sketched the graph of total revenue. Label this graph $TR(q)$.

FACT: The area under MR from 0 to q always gives $TR(q)$.

Getting from MC to TC will be harder. For one thing, in this scenario, the graph of MR is a line—we can easily compute, for example, the area under MR from 0 to 45 as the area of a *single* trapezoid. To compute the area under MC, however, we will have to break the region up into smaller pieces that approximate trapezoids. Further, the area under MC will give *an* anti-derivative of MC—we'll need to consider how VC and FC fit into this picture.

(c) Fill in the following table with the area under the MC graph on the indicated interval.

Interval	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55	55–60	60–65	65–70
Area under MC	25.5	18	13		10.5		18			48	63			123

(d) Add together the appropriate areas from part (c) to fill in the following table with the area under the MC graph from 0 to q.

q	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70
area under MC from 0 to q	0	25.5	43.5	56.5											

(e) We know that the table in part (d) gives values of a function that is an anti-derivative of MC. Moreover, this function goes through the origin. The anti-derivative of MC that goes through the origin is variable cost. On the axes in part (b), sketch and label the graph of $VC(q)$.

FACT: The area under MC from 0 to q always gives $VC(q)$.

(f) Fixed costs are \$100. Sketch and label the graph of $TC(q)$ on the axes in part (b).

(g) What quantity gives the largest possible profit?

(h) What is the largest possible profit?

(i) What is the largest quantity at which you won't be forced to take a loss?

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Math 112

Group Activity: Multivariable Functions

So far, in Math 111/112, we've investigated functions that have only one input variable, like $TR(q) = 25q - 0.5q^2$, which has one input variable, quantity q. For the remainder of the quarter, we'll study functions with more than one input variable. These are known as **multivariable functions.**

1. The balance in a savings account with continuously-compounded interest is given by the formula

$$A(P, r, t) = Pe^{rt},$$

where P is the principal (the amount initially invested), r is the annual interest rate expressed as a decimal, and t is time in years that the account has been accruing interest.

(a) Compute $A(5000, 0.06, 8)$ and write a sentence or two describing what it represents.

(b) Suppose you have exactly \$10,000 to use as principal and the only account available pays 4% interest, compounded continuously. Then the only variable that can change is time t.

For each of the following, translate into functional notation and compute.

i. the change in the balance from $t = 4$ to $t = 9$ years

ii. the average rate at which the balance changes (in dollars per year) from $t = 4$ to $t = 9$ years

(c) Suppose you've found an investment that promises 5% annual interest, compounded continuously, for a term of exactly 10 years. Then, the only variable that can change is the principal P.

i. You have \$1000 of your own to invest. Your friend offers to give you another \$500. How much would adding your friend's \$500 to the principal increase the pay-off amount of the investment?

ii. By how much will the pay-off amount increase if you increase the principal by one dollar: from P to $P + 1$?

2. In a certain math course, the final grade is determined by computing a weighted average of homework, participation, two midterm exams, and a final exam. The total number of points available for each component and its weighting is given in the following table.

Component	Points Earned	Points Possible	Weighting
Homework	h	600	15%
Participation	p	16	5%
Exam I	x	50	22%
Exam II	y	50	22%
Final	z	100	36%

At the end of the quarter, a student's total percentage is given by:

$$C(h,p,x,y,z) = \left(\frac{h}{600}\right)15 + \left(\frac{p}{16}\right)5 + \left(\frac{x}{50}\right)22 + \left(\frac{y}{50}\right)22 + \left(\frac{z}{100}\right)36,$$

which simplifies to

$$C(h,p,x,y,z) = 0.025h + 0.3125p + 0.44x + 0.44y + 0.36z.$$

This percentage is then converted into a grade as follows:

- If $C \geq 97$, then the student receives a 4.0 in the course.
- If $94 \leq C \leq 96$, then the student receives a 3.9 in the course.
- If $70 \leq C \leq 92$, then the student's grade is $0.1C - 5.5$.

(a) Terry earns 567 homework points, has a perfect participation score, and scores 43, 39, and 85 on the exams.

i. Compute Terry's total percentage: $C(567, 16, 43, 39, 85)$. (Round to the nearest whole number.)

ii. What grade does Terry receive in the course?

(b) Chris needs to earn at least a 2.7 in the course to keep a scholarship.

i. What total percentage C must Chris earn to receive a 2.7 in the course?

ii. Before the final, Chris has earned 576 homework points, 15 participation points, and midterm scores of 41 and 40. What must Chris earn on the final in order to receive a 2.7 in the course? (Round to the nearest whole number.)

(c) Pat requests a regrade on Exam II and receives 2 additional points on that exam. If no other scores change, how much will Pat's total percentage C increase?

(d) Which will lead to the largest increase in a student's total percentage: a 50-point increase in homework or a 5-point increase on the final exam?

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Math 112

Group Activity: Partial Derivative Practice

Compute the partial derivatives. Do not simplify.

1. $f(x,y) = x^5 - 4xy^2 + 7y^3 - 19$

   $f_x(x,y) =$ $f_y(x,y) =$

2. $z = x^2y + e^{3x} - \ln(y)$

   $\dfrac{\partial z}{\partial x} =$ $\dfrac{\partial z}{\partial y} =$

3. $f(x,y) = (x^2 - y^3)^4$

   $f_x(x,y) =$ $f_y(x,y) =$

4. $z = x\ln(xy)$

   $\dfrac{\partial z}{\partial x} =$ $\dfrac{\partial z}{\partial y} =$

5. $f(x,y) = 10 + 2x - 4y$

   $f_x(x,y) =$ $f_y(x,y) =$

6. $z = ye^x + x^2y$

   $\dfrac{\partial z}{\partial x} =$ $\dfrac{\partial z}{\partial y} =$

7. $g(q,r) = \dfrac{q^2 + 3r}{4q + 5r^2}$

   $g_q(q,r) =$ $g_r(q,r) =$

8. $z = x^3y^2 + 3y^3 - 6e^xy$

   $\dfrac{\partial z}{\partial x} =$ $\dfrac{\partial z}{\partial y} =$

9. $f(t,m) = te^m + t^2(m^2 + 2m)$

   $f_t(t,m) =$ $f_m(t,m) =$

10. $w = \dfrac{x^3y}{y+1}$

    $\dfrac{\partial w}{\partial x} =$ $\dfrac{\partial w}{\partial y} =$

11. $t = (s^2 + rs)(r + s^3)$

$\dfrac{\partial t}{\partial r} =$ $\dfrac{\partial t}{\partial s} =$

12. $g(u, v) = ve^{u^2 v}$

$g_u(u, v) =$ $g_v(u, v) =$

13. $p = x^3 y^2 + 3\ln(x^2 y) + 4x^2 - \dfrac{5x}{y}$

$\dfrac{\partial p}{\partial x} =$ $\dfrac{\partial p}{\partial y} =$

14. $h(u, w) = \dfrac{u + w}{u - w}$

$h_u(u, w) =$ $h_w(u, w) =$

15. $z = (m^2 p + p^2 m)(m + p)^3$

$\dfrac{\partial z}{\partial m} =$ $\dfrac{\partial z}{\partial p} =$