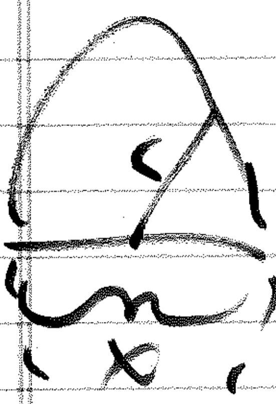
Optimization

Eg:

"norman window"

Perimeter 30 ft

max area



$r = \frac{1}{2}x$

$$A = xy + \frac{\pi \left(\frac{x}{2}\right)^2}{2}$$

Constraint

$$P = 30$$

$$2y + x + \frac{\cancel{2}\pi \left(\frac{x}{2}\right)}{\cancel{2}} = 30$$

$$2y + x + \frac{\pi}{2} x = 30$$

$$2y = 30 - \left(1 + \frac{\pi}{2}\right) x$$

$$y = 15 \cancel{\text{cm}} - \frac{1}{2}\left(1 + \frac{\pi}{2}\right) x$$

$$A = xy + \frac{\pi}{8}x^2$$

$$A = x\left(15 - \frac{1}{2}\left(1+\frac{\pi}{2}\right)x\right) + \frac{\pi}{8}x^2$$

$$= 15x - \frac{1}{2}\left(1+\frac{\pi}{2}\right)x^2 + \frac{\pi}{8}x^2$$

$$\frac{dA}{dx} = 15 - \left(1+\frac{\pi}{2}\right)x + \frac{\pi}{4}x = 0$$

$$15 = \left(1+\frac{\pi}{2}\right)x - \frac{\pi}{4}x$$

$$15 = \left(1+\frac{\pi}{4}\right)x$$

$$x = \frac{15}{1+\pi/4}$$

Check

$$\frac{d^2A}{dx^2} = \cancel{\pi/2} - \left(1+\frac{\pi}{2}\right) + \frac{\pi}{4}$$

$$\approx 1.0$$

$$< 0$$

$- \quad -$

$\frown$ rel max

$x \qquad A = ?$

$$y = 15 - \frac{1}{2}\left(1+\frac{\pi}{2}\right)x$$

$$A = xy + \frac{\pi}{8}x^2 \qquad \frac{15}{1+\pi/4}$$

Eg: 100 unit apt bldg

all rent at \$900 /month

but at \$1000 /month

only 90 rent

max revenue ?

x = # rented $\quad$ r = rent

Assume linear model

$x = f(r)$

r	x
900	100
1000	90

$$\frac{\Delta x}{\Delta r} = \frac{-10}{100}$$

$$x - 100 = \frac{-1}{10}(r - 900)$$

$$x = 100 - \frac{r-900}{10}$$

demand fcn

$$y - b = m(x - a)$$

$$\frac{-1}{10}(r$$

revenue

$$R(r) = r \cdot x$$

$$= 100r - \frac{r-900}{10} \cdot r$$

$$r \cdot x = r\left(100 - \frac{r-900}{10}\right)$$

$$R(r) = 100r - \frac{r^2}{10} + 90r$$

$$= 190r - \frac{r^2}{10}$$

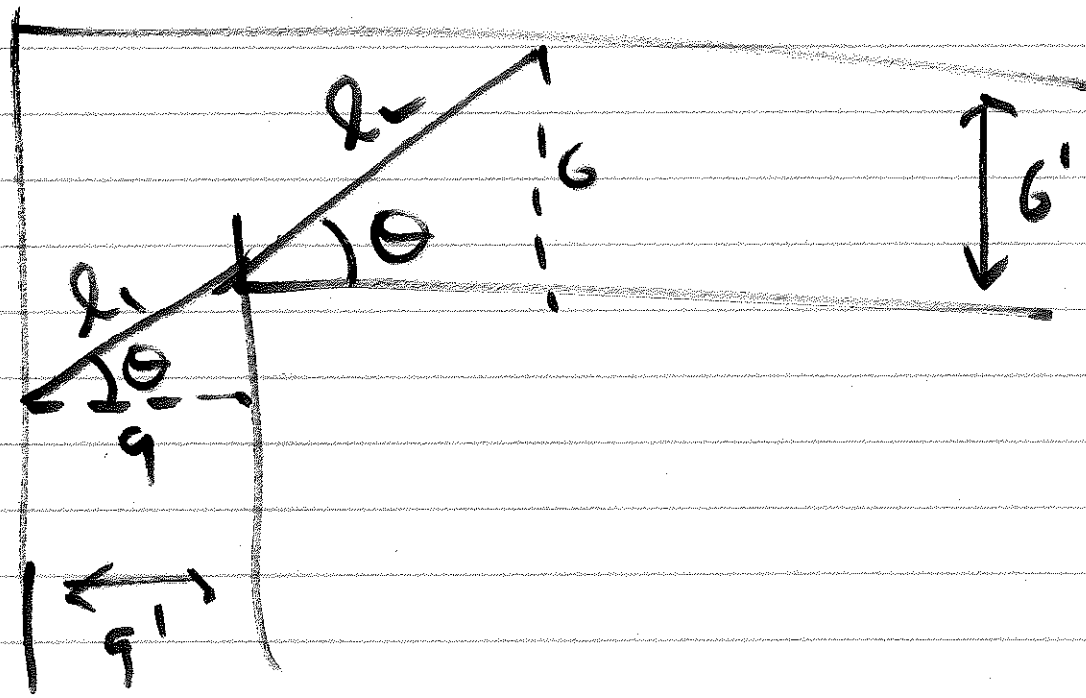
$$R'(r) = 190 - \frac{r}{5} = 0$$

$$r = 950$$

$$R''(r) = -\frac{1}{5}$$

2nd deriv test

rel max



~~max~~ l min l ✓

$$l = l_1 + l_2$$

similar $\triangle$'s

$$\frac{6}{l_2} = \sin\theta$$

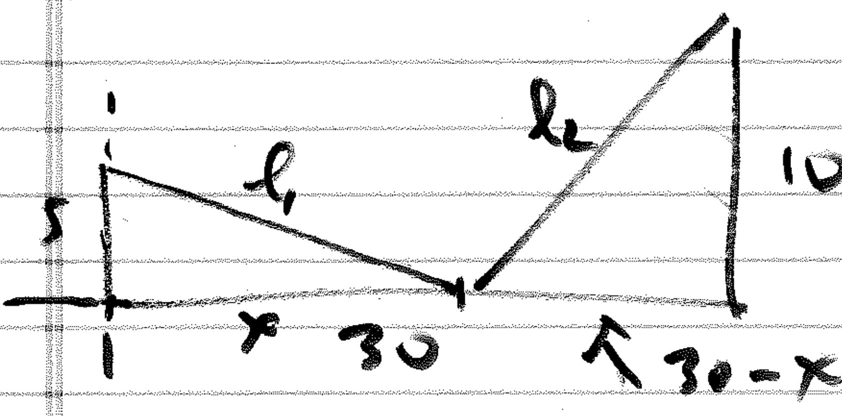
$$\frac{9}{l_1} = \cos\theta$$

$$l_2 = \frac{6}{\sin\theta} = 6\csc\theta$$

$$l_1 = \frac{9}{\cos\theta} = 9\sec\theta$$

$$l = 6\csc\theta + 9\sec\theta$$

$$0 \le \theta \le \pi/2$$



$$\min\ l = l_1 + l_2$$

$$x^2 + 5^2 = l_1^2$$

$$(30-x)^2 + 10^2 = l_2^2$$

$$\frac{(x+3)(x-2)}{x(x-2)^2}$$