Optimization

"max/min"

Final - Kane 110

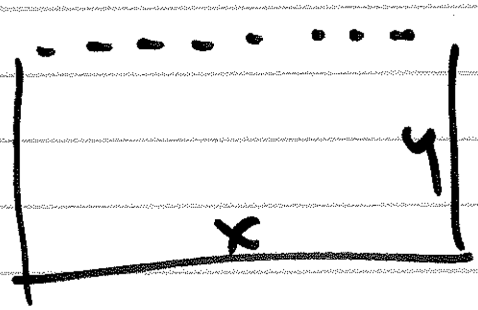
Seating chart

11:30 12/11

4:20

Ex: Fence

rect



300 ft^2

3 wood walls \$10 ft

1 chain link \$5 ft

$$C = 15x + 20 \cdot \frac{300}{x}$$

$$C = 15x + \frac{6000}{x}$$

objective

min C

$x > 0$

$$\frac{dC}{dx} = 15 - \frac{6000}{x^2}$$

$$0 = 15 - \frac{6000}{x^2}$$

$$0 = 15x^2 - 6000$$

$$400 = x^2$$

$$20 = x$$

dimensions
that give min cost?

- decision variables

$xy = A = 300$ ✓

$C = 5x + 10(y + x + y)$

$C = 15x + 20y$ ←

- constraint

1 variable

$y = \frac{300}{x}$ $y = \frac{300}{20}$

20 × 15

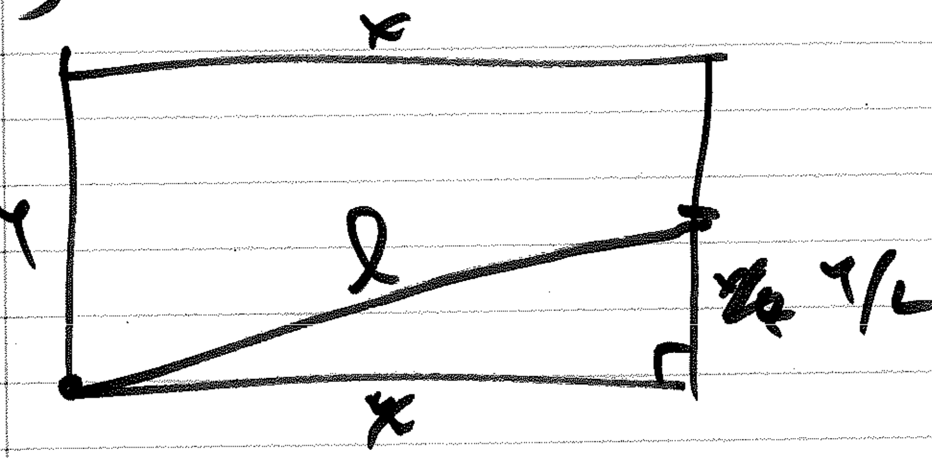
check

$$\frac{d^2C}{dx^2} = +\frac{12000}{x^3}$$

> 0 at $x = 20$

$+ +$ min

Eg:



$A = 2 \text{ in}^2$

$\min \ l$

$xy = 2$

$l^2 = \cancel{y^2 + \left(\frac{x}{2}\right)^2}$

$\quad = x^2 + (y/2)^2$

$$\ell = \sqrt{x^2 + \frac{y^2}{4}}$$

$$x = \frac{2}{y}$$

$$\ell = \sqrt{\frac{4}{y^2} + \frac{y^2}{4}}$$

$y > 0$

$$\frac{d\ell}{dy} = \frac{1}{2\sqrt{4/y^2 + y^2/4}} \cdot \underline{\underline{\left(\frac{-8}{y^3} + \frac{y}{2}\right)}}$$

$$\frac{4}{y^2} + \frac{y^2}{4} = 0 \quad ?$$

$$\left(\frac{2}{y}\right)^2 + \left(\frac{y}{2}\right)^2 \neq 0$$

$l^2 = x^2$

$$l^2 = \frac{4}{y^2} + \frac{y^2}{4}$$

$$2l\frac{dl}{dy} =$$

$$2y^3 \quad -\frac{8}{y^3} + \frac{y}{2} = 0 \quad 4y^3$$

$$-16 + y^4 = 0$$

$$y = 2$$

min

$-$ $+$

1 2 3

$$\frac{dh}{dy} = \frac{1}{2\sqrt{m}}$$

pos

$$-\frac{8}{y^3} + \frac{y}{2}$$

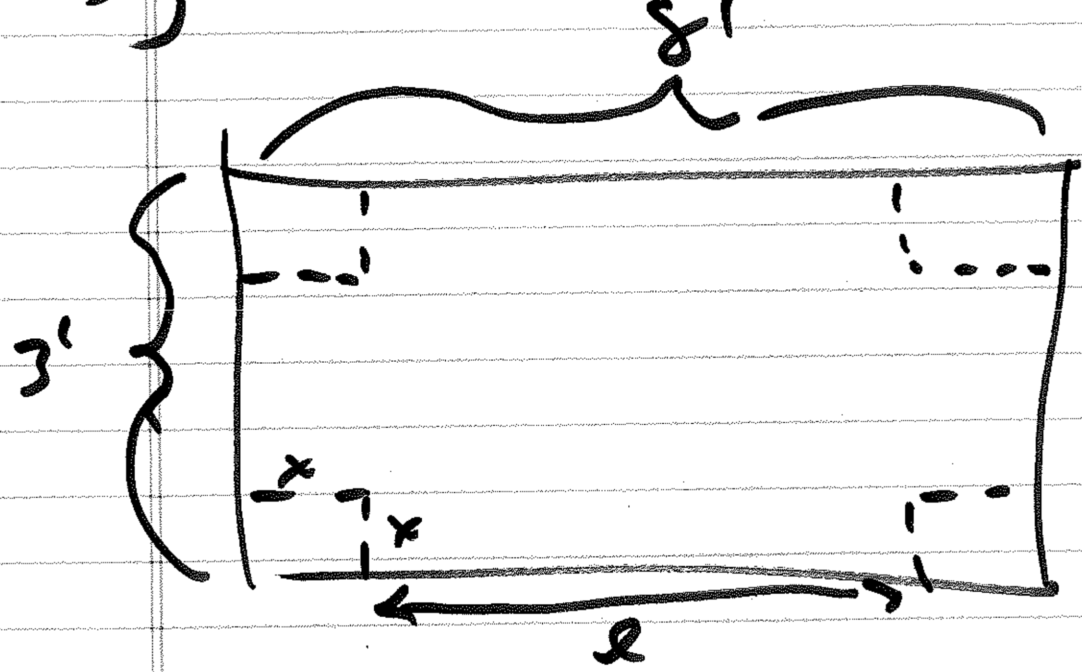
$-8 + \frac{1}{2}$

at $y = 1$

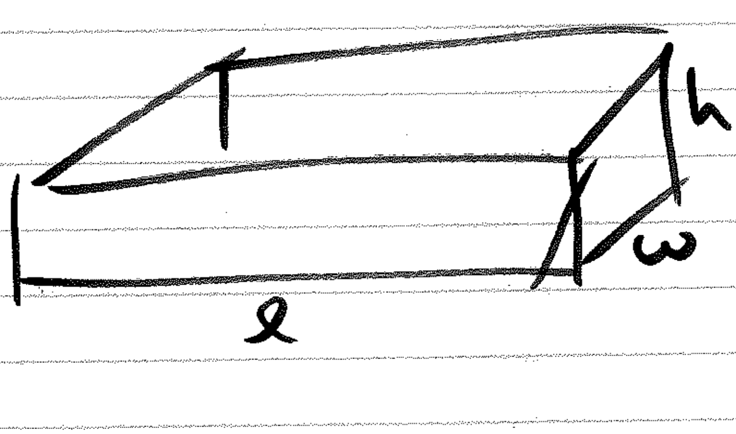
$y = 3$

$$-\frac{8}{27} + \frac{3}{2} > 0$$

Eg: make a box



max vol



$V = lwh$

$h = x$

$l = 8 - 2x$

$w = 3 - 2x$

$V = x(3-2x)(8-2x)$

$0 \le x \le 3/2$

$3 - 2x \ge 0$

$$V = 24x - 22x^2 + 4x^3$$

$$\frac{dV}{dx} = 24 - 44x + 12x^2$$

$$3x^2 - 11x + 6 = 0$$

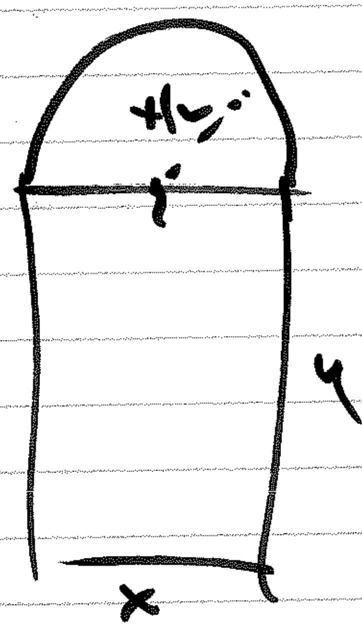
$$b^2 - 4ac$$

$$121 - 72 = 49$$

$$\frac{11 \pm 7}{6} \qquad 3, \frac{2}{3}$$

x	V
0	0
$\frac{2}{3}$	$\leftarrow$ max
$\cancel{3}$	
$3/2$	0

Eg:



$P = 30$ ft

max area

$$A = \cancel{x} \, xy + \pi\left(\frac{x}{2}\right)^2$$

$$30 = y + x + y + \frac{2\pi\left(\frac{x}{2}\right)}{2}$$