Marquis de l'Hôpital

l'Hôpital's Rule

$$\lim_{x \to a} \frac{f(x)}{g(x)} \qquad \frac{0}{0}$$

indeterminate forms

$$f(a) = g(a) = 0$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)} \frac{\frac{1}{x-a}}{\frac{1}{x-a}}$$

$$\lim_{x\to a} \frac{\frac{f(x)-f(a)}{x-a}}{\frac{g(x)-g(a)}{x-a}} = \frac{f'(a)}{g'(a)}$$

~~Thm~~:

Eg: $\lim_{x\to 0} \frac{\sin x}{x} \quad \frac{0}{0}$

$$= \lim_{x\to 0} \frac{\cos x}{1} = \frac{\cos(0)}{1}$$

$$= 1$$

Thm (l'Hôpital)

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

f, g diff

① $f(a) = g(a) = 0$ $\qquad \frac{0}{0}$

② $\lim_{x \to a} f(x) = \pm\infty$ $\qquad \frac{\infty}{\infty}$

$\lim_{x \to a} g(x) = \pm\infty$

then $$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Eg: $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$ $\frac{\infty}{\infty}$

$\overset{\text{l'Hôp}}{=} \lim_{x\to\infty} \frac{1/x}{1/2\sqrt{x}}$

$= \lim_{x\to\infty} \frac{2\sqrt{x}}{x}$

$= \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0$

Eg: $\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

$\infty - \infty$

indeterminate form

$$\frac{x-1}{x-1}\frac{1}{\ln x} - \frac{1}{x-1}\frac{\ln x}{\ln x}$$

$$\lim_{x\to 1} \frac{x-1-\ln x}{\ln x\,(x-1)} \qquad \frac{0}{0}$$

$$\overset{\text{l'Hôp}}{=} \lim_{x\to 1} \frac{1-1/x}{\frac{1}{x}(x-1)+\ln x} \qquad \frac{x}{x}$$

$$= \lim_{x \to 1} \frac{x-1}{x-1+x\ln x} \qquad \frac{0}{0}$$

l'hôp

$$= \lim_{x \to 1} \frac{1}{1+\ln x + x \cdot \frac{1}{x}}$$

$$= \lim_{x \to 1} \frac{1}{2+\ln x} = \frac{1}{2}$$

Eg: $\lim_{x \to 0^+} x^{\sin x}$ $\qquad x \geq 0$

$f(x) = x^{\sin x}$

$f(0) = 0^0$ ind. form

$$\lim_{x \to 0^+} \ln\left(x^{\sin x}\right) = \ln A$$

$$e^{\ln A} = A$$

if $\lim x^{\sin x} = A$

$$\lim \ln x^{\sin x} = \ln A$$

remember - un log at end

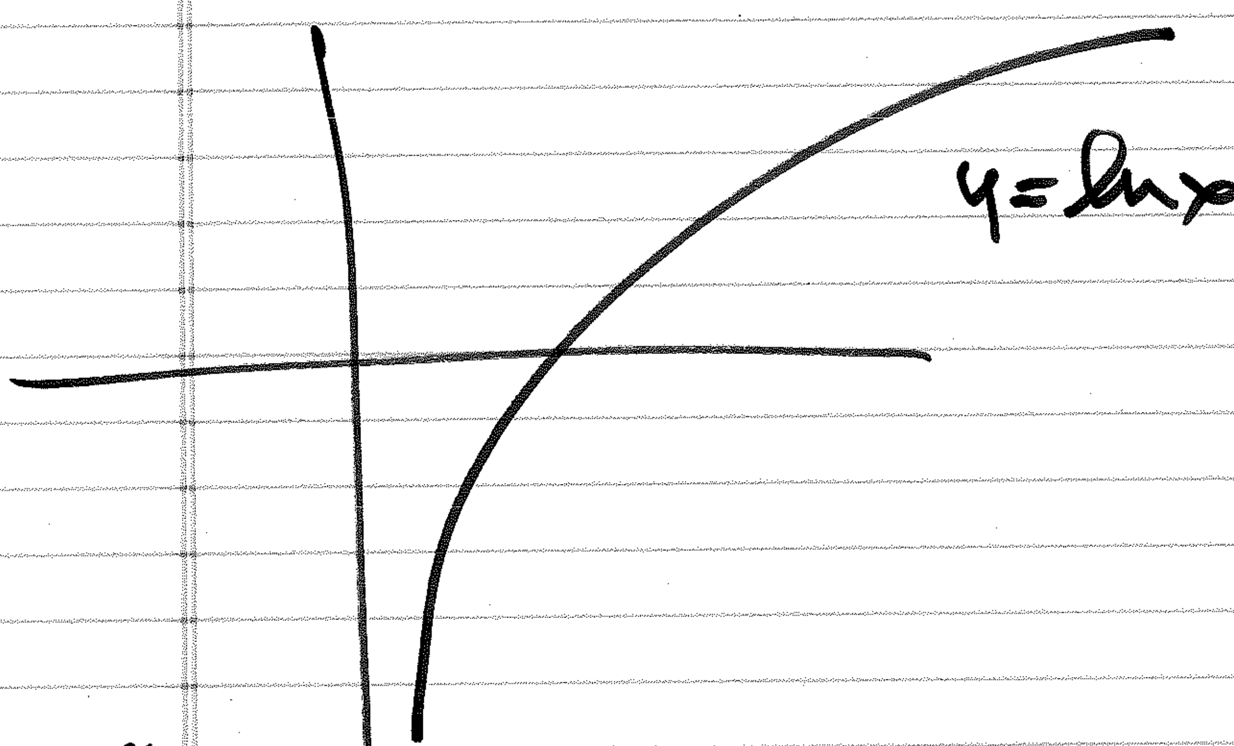
$$\lim_{x \to 0^+} \sin x \cdot \ln x$$

$$0 \ , \ -\infty$$

$$\lim_{x \to 0^+} \frac{\ln x}{\csc x} \qquad 0 \cdot \infty \qquad \frac{1}{\sin x}$$

$$\sin x \cdot \ln x = \frac{\ln x}{1/\sin x}$$

$$\lim_{x\to 0^+} \frac{\ln x}{\csc x} \qquad \frac{-\infty}{\infty}$$



l'Hôp

$$= \lim_{x\to 0^+} \frac{1/x}{-\csc x \cot x}$$

$$\frac{\frac{1}{x}}{-\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}} \quad \cancel{\frac{x}{\sin x}}\ \frac{x}{x}\cdot\frac{\sin^2 x}{\sin^2 x}$$

$$\lim_{x\to 0^+} -\frac{\sin^2 x}{x\cos x} \qquad \frac{0}{0}$$

l'hôp

$$= \lim_{x\to 0^+} -\frac{2\sin x\cos x}{\cos x - x\sin x} \qquad \frac{0}{1}$$

$$= \lim_{x\to 0^+} -0$$

$$\begin{cases} 0 = \ln A \\ e^0 = A \\ 1 = A \end{cases}$$

Graphing

$$g(x) = \frac{48}{x^2+12}$$

① asymptotes

no vert

$$\lim_{x \to \pm\infty} g(x) = 0$$

$y = 0$

② inc / dec

③ concavity

$$g(x) = 48\left(x^2+12\right)^{-1}$$

$$g'(x) = -48\left(x^2+12\right)^{-2} \cdot 2x$$

$$= -\frac{96x}{\left(x^2+12\right)^2}$$

crit val $x=0$

rel max

inc | dec

-1 0 1

$$g'(-1) = \frac{96}{13^2}$$

$$g'(1) = -\frac{96}{13^2}$$

$$f(0) = \frac{48}{12} = 4$$