Analysis $y = f(x)$

$f(x) = 2 - \frac{3}{x} + \frac{4}{x^2}$

Domain $x \neq 0$

Asymptotes

hori $\lim_{x \to \pm\infty} f(x)$

vert

undefined : $x = 0$

$\lim_{x \to 0^+} f(x)$

$\lim_{x \to 0^-} f(x)$

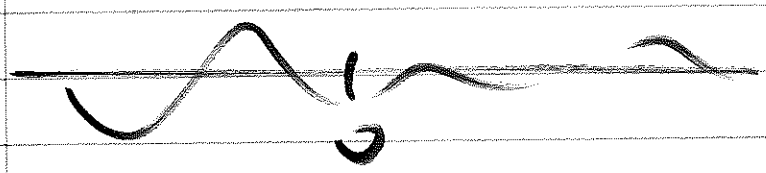
horiz

$\lim_{x \to \pm\infty} f(x) = 2$

$y = 2$

vert $x = 0$

$\lim_{x \to 0^+} 2 - \frac{2}{x} + \frac{4}{x^2}$



$\lim_{x \to 0^+} \frac{2x^2 - 3x + 4}{x^2}$ f +

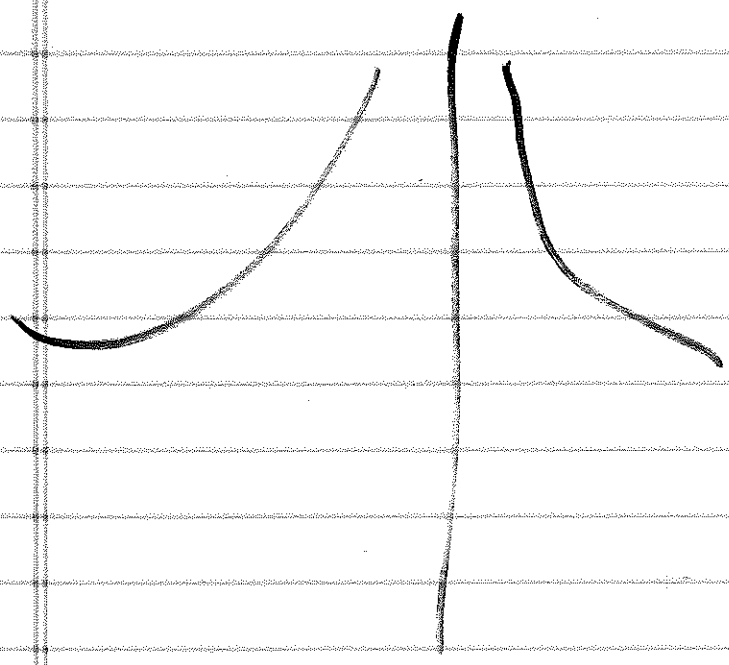
zeros of top

$$x = \frac{3 \pm \sqrt{9-3x}}{4}$$

no zeros!

pos at $x=0$

always pos



$$f(x) = 2 - \frac{3}{x} + \frac{4}{x^2}$$

Inc !

$f'(x) > 0$ $\quad x = ?$

$$f(x) = 2 - 3x^{-1} + 4x^{-2}$$

$$f'(x) = 3x^{-2} - 8x^{-3}$$

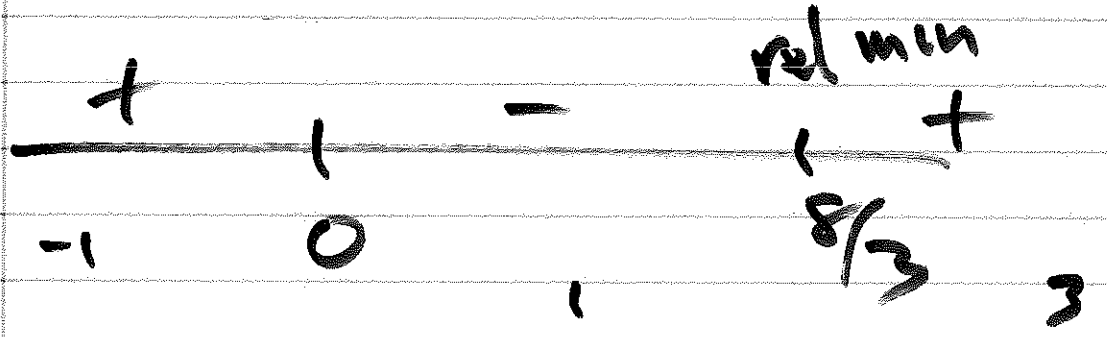
$$= \frac{3x}{x^2 x} - \frac{8}{x^3}$$

$$= \frac{3x - 8}{x^3}$$

$x = 5/3$, 0

$5/3$: zeros

0: undef, denom $= 0$



x	$f'(x)$
-1	$-11/-1 > 0$
1	$-5/1 < 0$
3	$1/27 > 0$

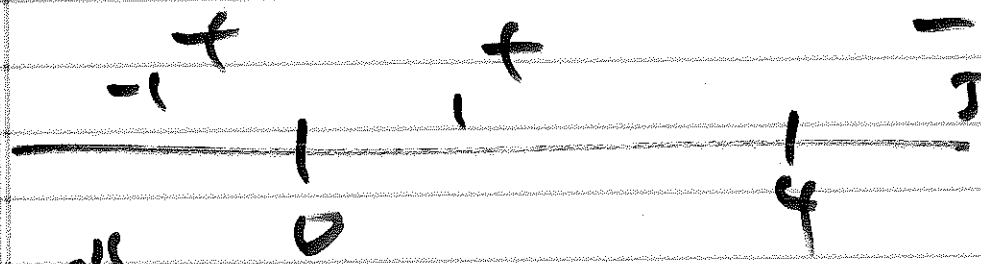
$x < 0$ or $x > 5/3$

$(-\infty, 0) \cup (5/3, \infty)$

Canc down?

$$f''(x) = -6x^{-3} + 24x^{-4}$$

$$= \frac{-6x + 24}{x^4} \qquad x = 0, 4$$



x	f''
-1	30/1
1	18/1
5	-6/625

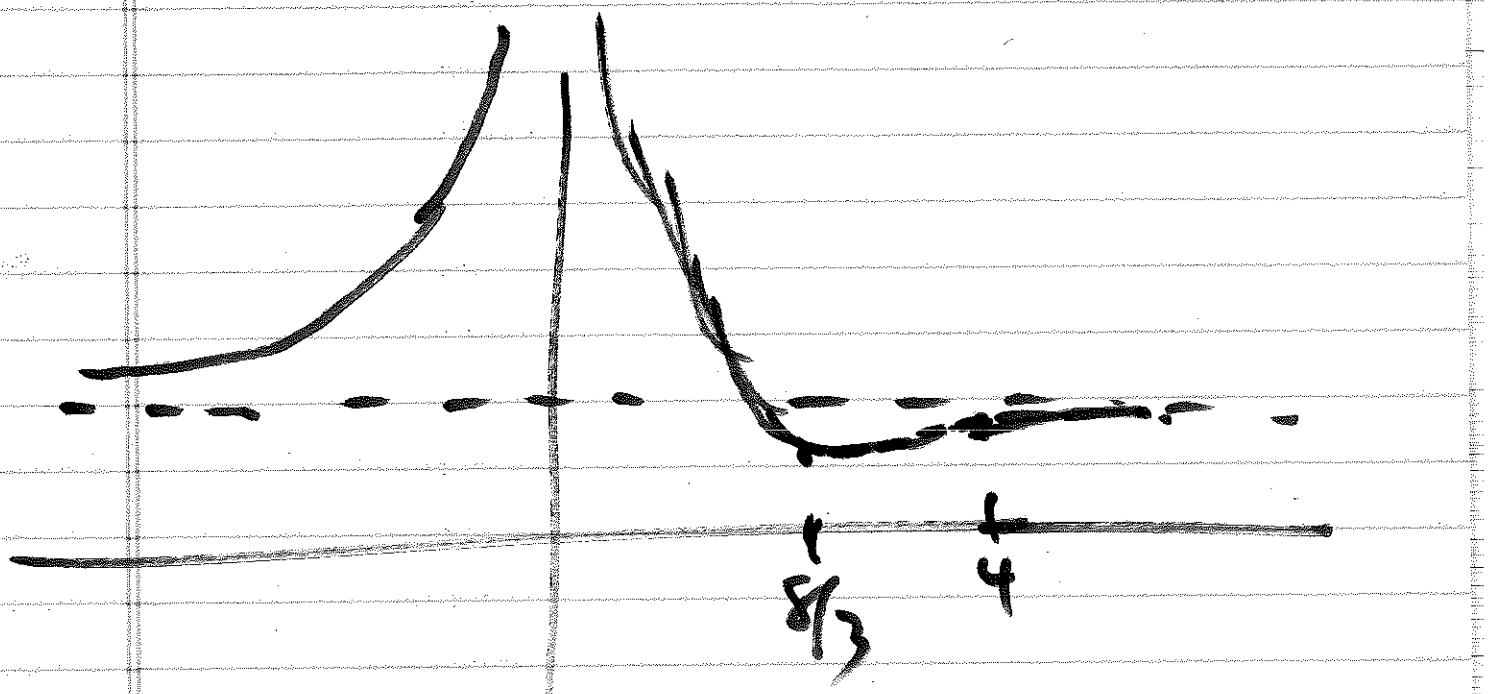
$x > 4$

$(4, \infty)$

Pt of I at $x = 4$

x	$y = f(x)$
$\frac{8}{3}$	~
4	~

$$f(x) = \frac{2x^2 - 3x + 4}{x^2}$$


1/3
1/4

Ex: $f(x) = \ln(x^2+1)$

crit vals

$$f'(x) = \frac{1}{x^2+1} \cdot 2x$$

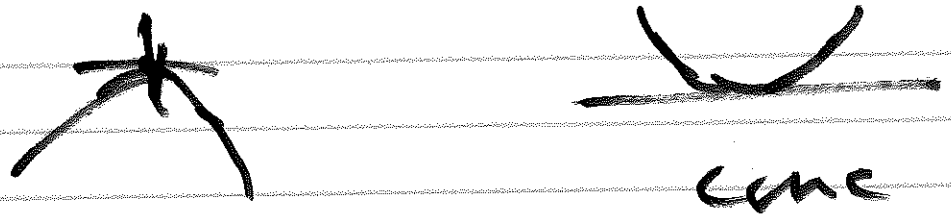
$$= \frac{2x}{x^2+1}$$

$x = 0$ min or max?

$$f''(x) = \frac{2(x^2+1) - 2x \cdot 2x}{(x^2+1)^2}$$

$$f''(0) = \frac{2 \cdot 1}{1} > 0$$

rel min $\cap$ $\cup$ $(- -)$ $(+ +)$



Thm: If $f'(a) = 0$
and $f''(a) > 0$
then $y = f(x)$ has a
rel min at $x = a$

Similarly for $f''(a) < 0$
rel max

Eg: $p(t) = e^{\sin t}$

$[0, 7]$

$0 \le t \le 7$

crit vals

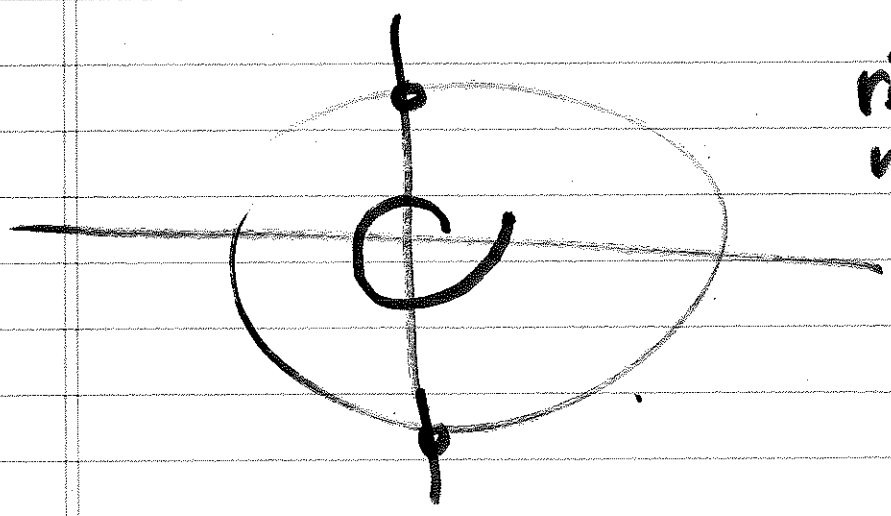
$p'(t) = \overbrace{e^{\sin t}}^{u} \cdot \overbrace{\cos t}^{v}$

$e^{\sin t}$ never 0

$\cos t = 0$ $\quad t = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{\pi}{2}$: rel max

$\frac{3\pi}{2}$: rel min



max / min !

$$p''(t) = \underbrace{e^{\sin t}\cos t}_{u'} \cdot \underbrace{\cos t}_{v}$$

$$+ e^{\sin t} \cdot -\sin t$$

$$= e^{\sin t}\left(\cos^2 t - \sin t\right)$$

$$p''(\pi/2) = +(0 - 1) < 0$$

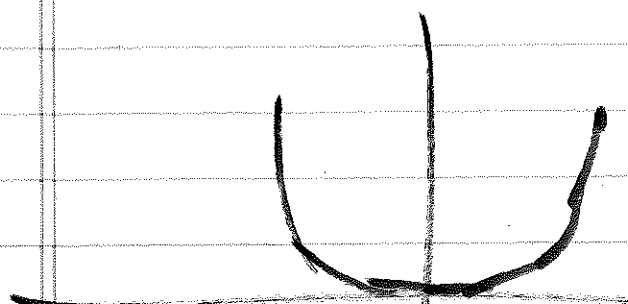
$$p''\left(\frac{3\pi}{2}\right) = +\left(0 - (-1)\right) > 0$$

$f(x) = x^4$

$f'(x) = 4x^3$ & $x = 0$

$f''(x) = 12x^2$

$\boxed{f''(0) = 0}$ Inconclusive



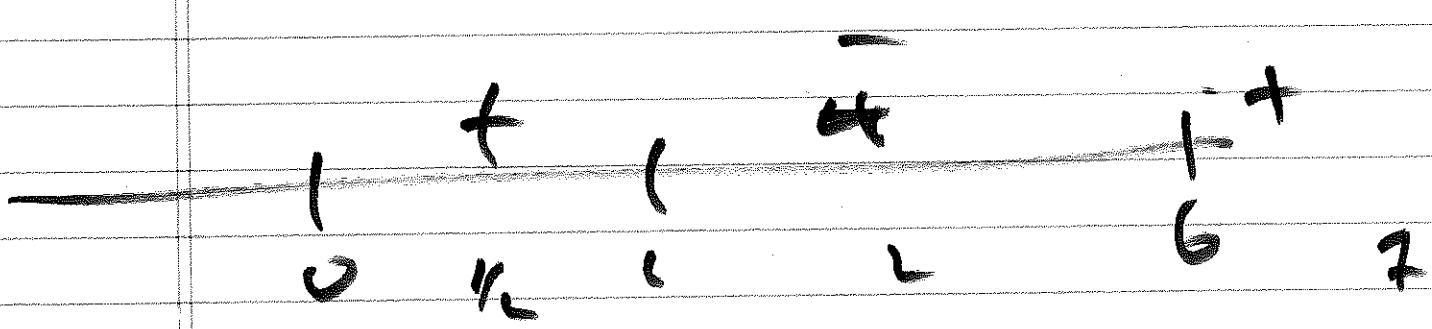
Ex: $f(x) = \underbrace{\sqrt{x}}_{u}\underbrace{\left(3x^2 - 35x + 90\right)}_{v}$

$$f'(x) = \frac{1}{2\sqrt{x}}\left(3x^2 - 35x + 90\right) + \frac{\sqrt{x}\left(6x - 35\right)2\sqrt{x}}{2\sqrt{x}}$$

$$= \frac{3x^2 - 35x + 90 + 2x\left(6x - 35\right)}{2\sqrt{x}}$$

$$= \frac{15x^2 - 105x + 90}{2\sqrt{x}}$$

$x = 0$, $1, 6$ $\qquad \dfrac{15(x-1)(x-6)}{2\sqrt{x}}$



domain $x \geq 0$