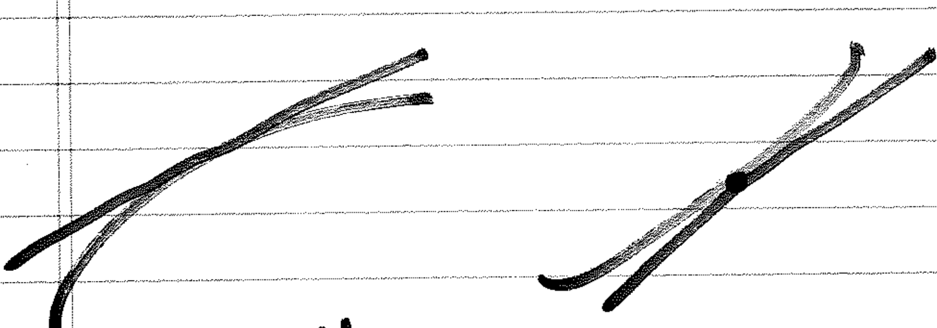
Applications
- Linear Approx
- Analysis
- Optimization

Analysis
rel min / max
crit nums

$$f' \begin{cases} \text{increasing / decreasing} \\ f(a+h) > f(a) \\ h>0 \end{cases}$$



$f'>0$ $f''<0$

inflection points
concave down
up

f''

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eg: $f(x) = 2x^3 + 3x^2 - 12x$

0^{th} deriv $f(x)$

x, y intercepts

asymptotes

1st deriv

$f'(x) = 6x^2 + 6x - 12$

Facts: $f'(x) = 0$ crit vals
rel min/max

$f'(x) > 0$ inc

$f'(x) < 0$ dec

sign analysis

$6x^2 + 6x - 12 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

rel max   rel min

$+$ $-$ $+$

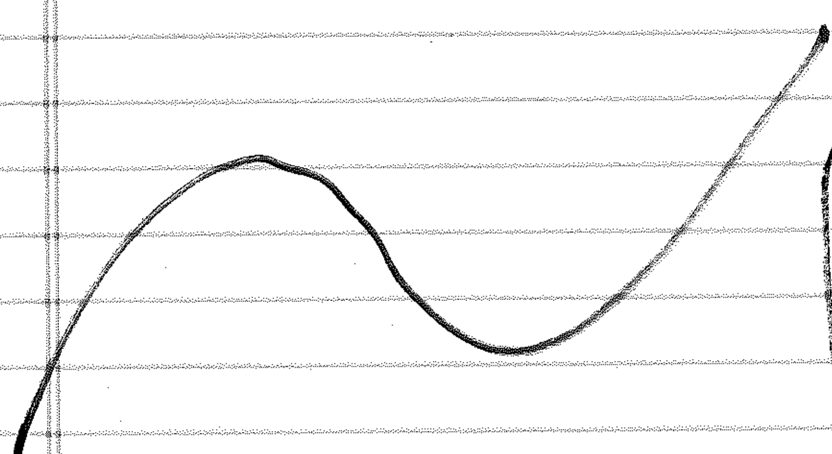
-2 1

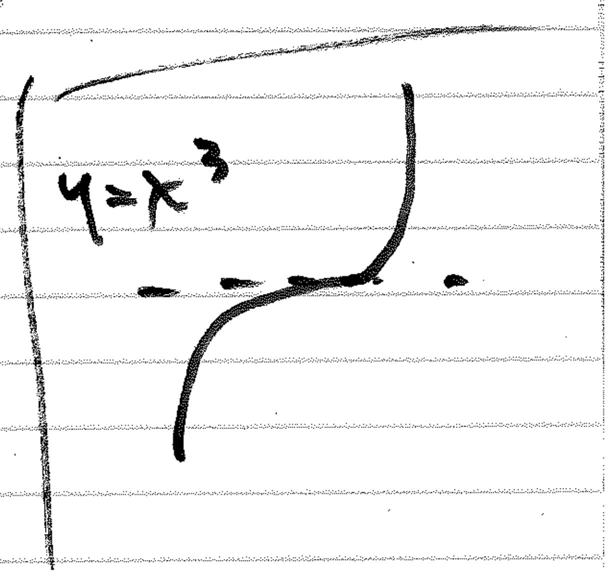
$f'(-3) = 24$ $\cancel{f''}$ $f'(2) = 24$

$f'(0) = -12$

$f(x)$ is inc
when $x < -2$ or $x > 1$

$f(x)$ is dec
when $-2 < x < 1$





2^{nd} deriv

$f''(x) > 0$ conc up

$f''(x) < 0$ conc down

$\overset{--}{\frown}$ $\qquad$ $\overset{++}{\smile}$

Points of Infl

Crit vals $\leftarrow$

$f'(x) = 6x^2 + 6x - 12$

$f''(x) = 12x + 6 = 0$

$x = -1/2$

Sign analysis

P of I

$-$ $\quad\quad$ $+$

$-\frac{1}{2}$

$f''(-1) = -6$ $\quad\quad$ $f''(0) = 6$

$f(x)$ conc up $\quad x > -\frac{1}{2}$

— down $\quad x < -\frac{1}{2}$

Sign must change for P of I

Eg:

$g(x) = x^4 \boxed{e^{-x}}$

1st $\quad g'(x) = \cancel{(4x^3 + x^4)e}$

$= 4x^3 e^{-x} + x^4 e^{-x} \cdot -1$

$= (4x^3 - x^4)e^{-x} = 0$

$4x^3 - x^4 = 0 \qquad e^{-x} > 0$

$x^3(4 - x) = 0$

$x = 0, 4$

$-$ 0 $+$ 4 $-$

$g'(-1)$ $g'(1)$ $g'(5)$

x	$f'(x)$
1	$3e^{-1}$
-1	$-5e$
5	neg

$(4 \cdot 125 - 625)e^{-5}$

$f(x)$ inc $(0,4)$

$0 < x < 4$

dec $(-\infty, 0) \cup (4, \infty)$

rel min $(0,0)$ point!

rel max $(4, 256e^{-4})$

2^{nd}

$$f''(x) = (12x^2 - 4x^3)e^{-x}$$
$$- (4x^3 - x^4)e^{-x}$$

$$= (12x^2 - 8x^3 + x^4)e^{-x} = 0$$

$$x^2(x^2 - 8x + 12) = 0$$

$$\boxed{x^2(x-6)(x-2) = 0}$$

$x = 0, 2, 6$

P of I   P of I

$+$ $+$ $-$ $+$

0   2   6

$f''(-1)$ $f''(1)$ $f''(3)$ 7

-1 $1 \cdot (-1-6)(-1-2)$

$-$ $-$

1 $1 \cdot (1-6)(1-2)$

$-$ $\quad$ $-$

3 $9 \cdot (3-6)(3-2)$

$-$ $\quad$ $+$

5 ~~25 · (5 −~~

$49 \cdot (7-6)(7-2)$

$+$ $\quad$ $+$

Eg: $f(x) = x^{x^2}$

1st

$y = x^{x^2}$ $\quad \ln x^{x^2}$

$\ln y = x^2 \cdot \ln x$

$\frac{y'}{y} = 2x \ln x + x^2 \cdot \frac{1}{x}$

$y' = x^{x^2} \cdot x \, (2\ln x + 1)$

$(1 + 2\ln x) = 0$

$x = 0$

$$1 + 2\ln x = 0$$

$$e^{\ln x} = e^{-1/2}$$

$$x = \frac{1}{\sqrt{e}}$$

$$x = e^{-1/2}$$