Linearization

Eg: approx $\sqrt[4]{80}$

① $f(x) = x^{1/4}$ $\boxed{y = x^{1/4}}$

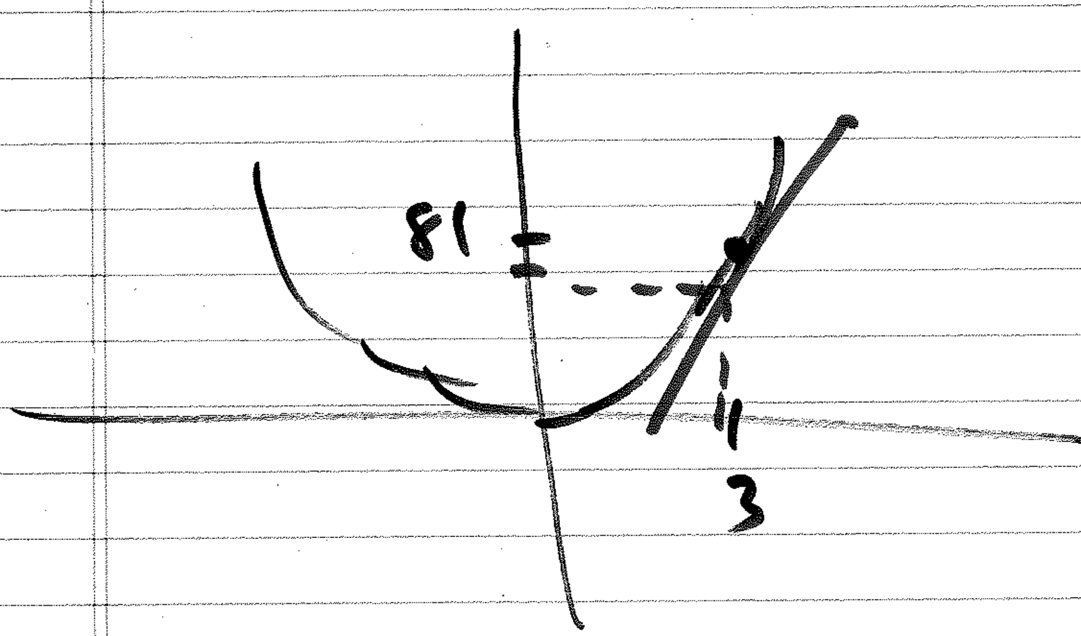
$a = 81 = 3^4$

② $g(x) = x^4$

$\boxed{y = x^4}$ $80 = x^4$

$(3, 81)$

approx x 80



$y = x^4 \qquad (3, 81)$

$$\frac{dy}{dx} = 4x^3 \Big|_{x=3} = 108$$

$$y - 81 = 108(x-3)$$

$$L(x) = 81 + 108(x-3)$$

approx $\sqrt[4]{80}$

$y = 80$ approx x

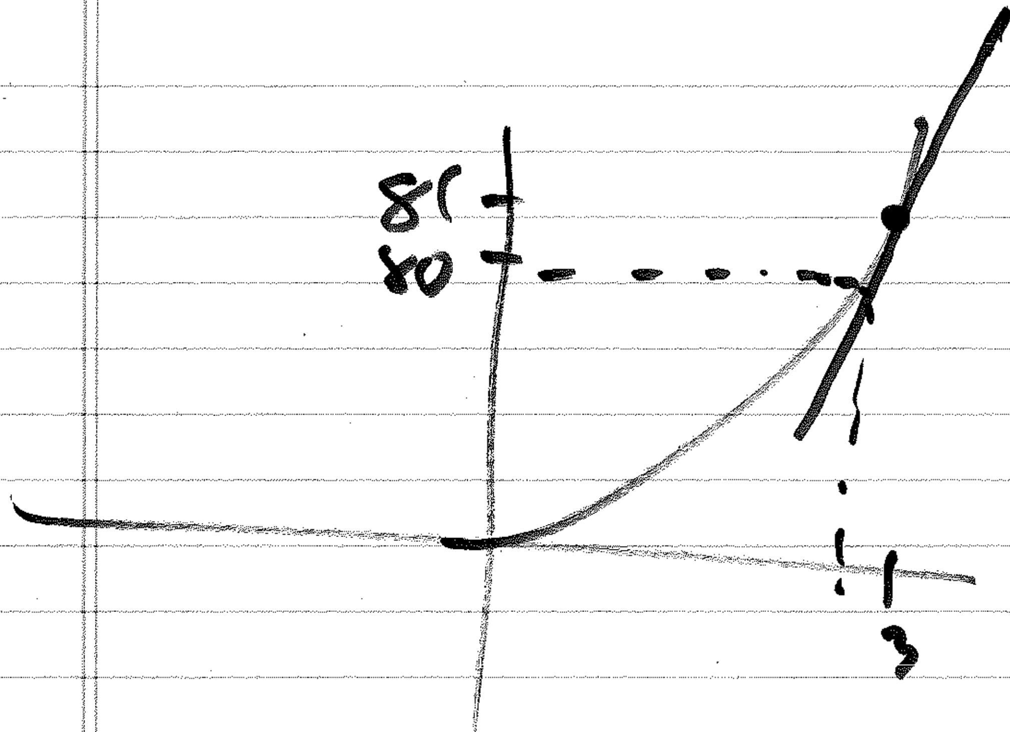
$80 = 81 + 108(x-3)$

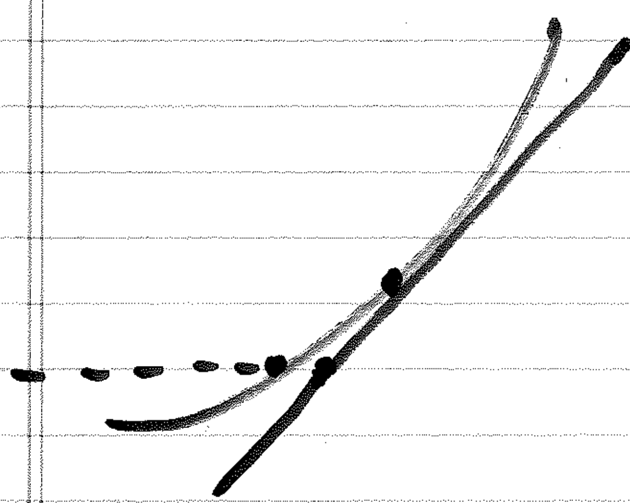
$-1 = 108(x-3)$

$-\frac{1}{108} = \underbrace{x-3}_{\Delta x}$

$x \approx 3 - \frac{1}{108}$

over/under

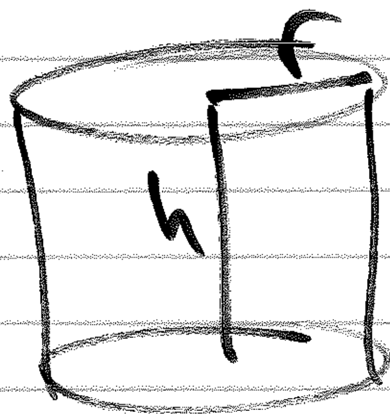




normally $\left[\begin{array}{ll} y'' > 0 & \text{under} \\ y'' < 0 & \text{over} \end{array}\right]$

- Optimization
  max/min

Eg: soup can

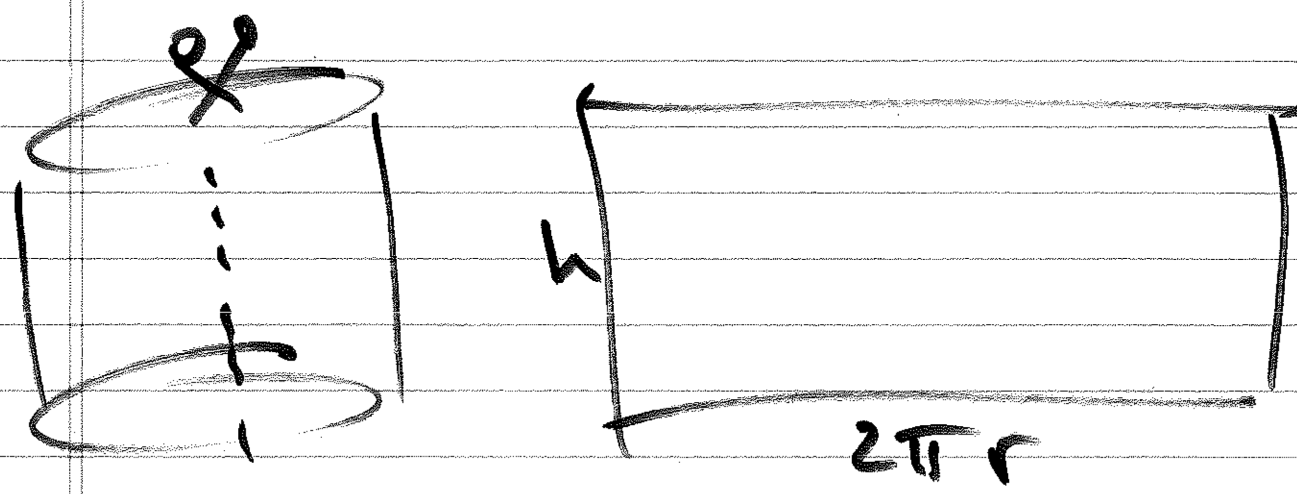


$V = 800$ cc
cm^3

dimensions of cheapest can

min SA
st $V = 800$

$$SA = 2 \cdot \pi r^2 + 2\pi r h$$



Constraint

$$V = \pi r^2 h$$

$$\pi r^2 h = 800$$

$$h = \frac{800}{\pi r^2}$$

$$A = 2\pi r^2 + 2\pi r \cdot \frac{800}{\pi r^2}$$

min $\boxed{A = 2\pi r^2 + \frac{1600}{r}}$

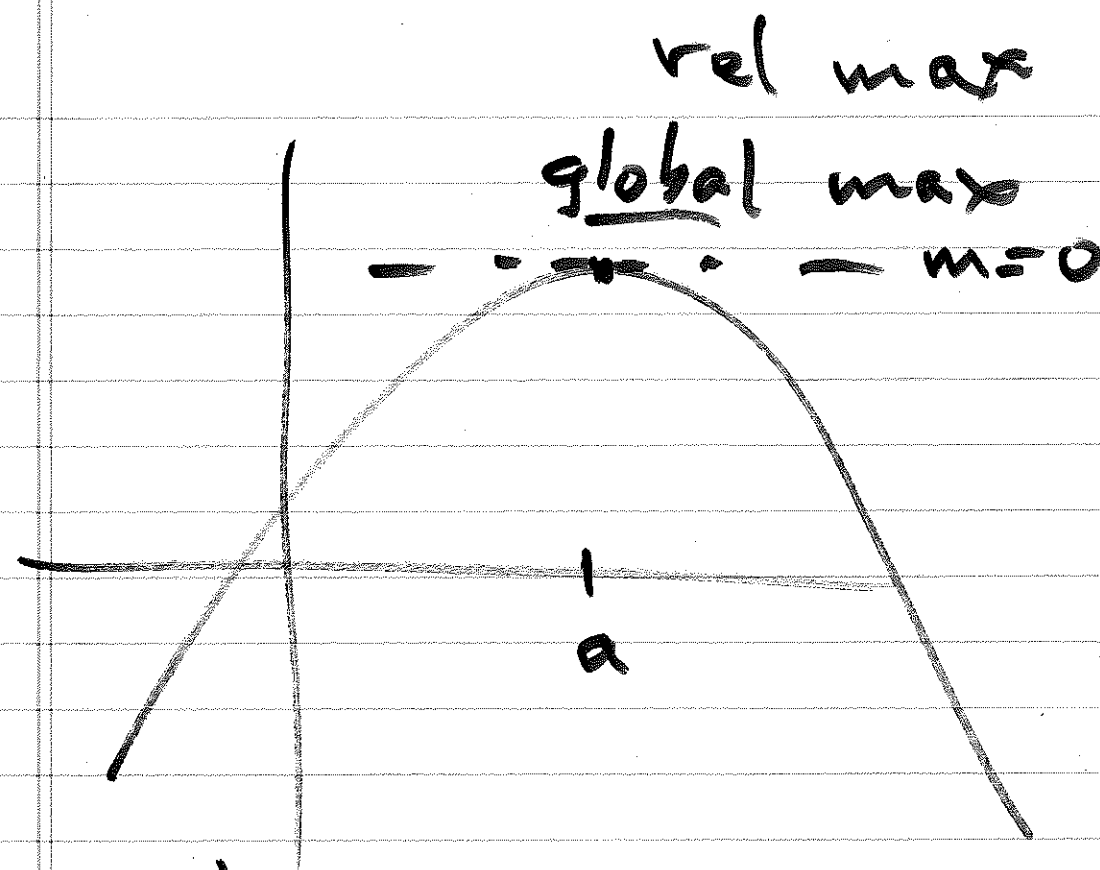
st $r > 0$

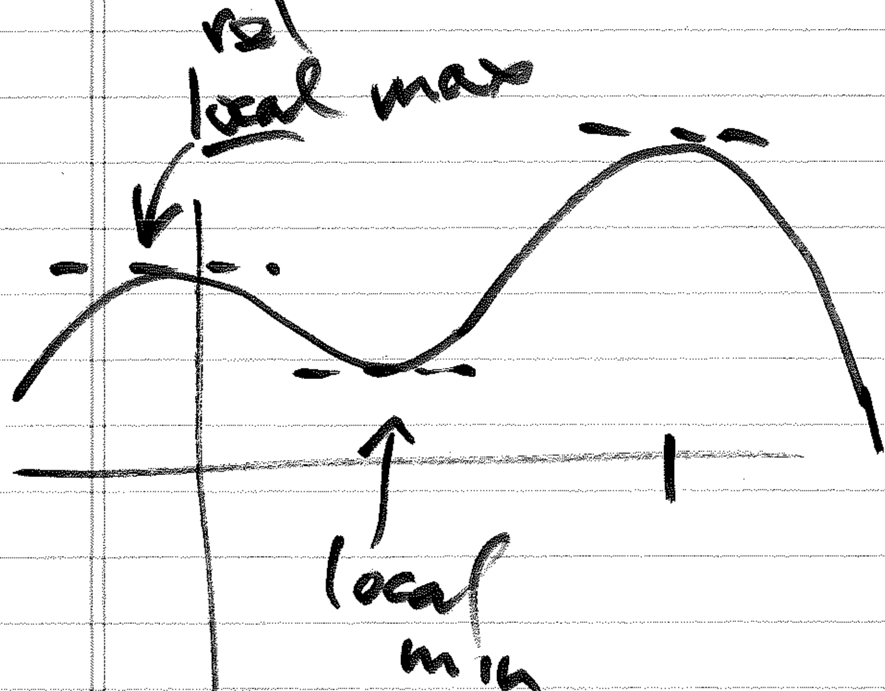
<u>Thm</u> (Fermat)

If $y = f(x)$ is diff'ble at $x = a$ and has a min/max at $x = a$ then $f'(a) = 0$

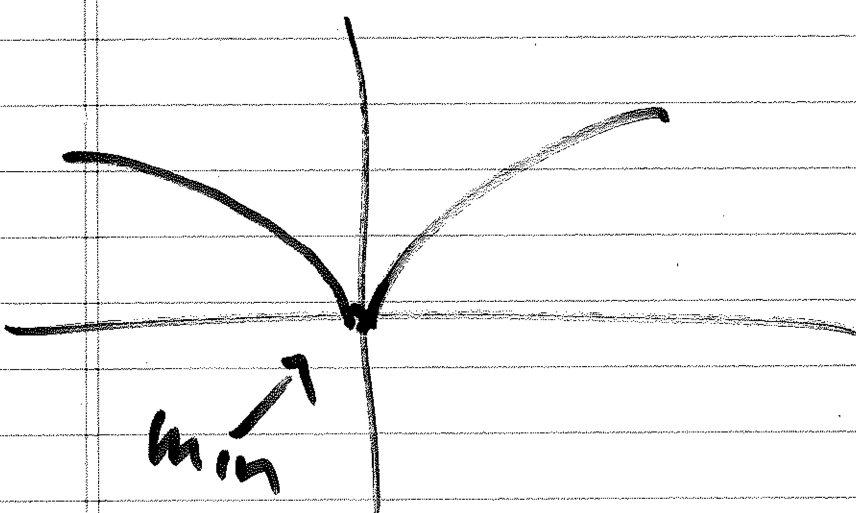
<u>critical</u> <u>numbers</u>

$f'(x) = 0$ or $f'(x)$ undef


rel max
global max
m=0
a


rel
local max
local min

$y = x^{2/3}$



$$\frac{dy}{dx} = \frac{2}{3} x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$$

$$\frac{dA}{dr} = r^2\left(4\pi r - \frac{1600}{r^2}\right) = 0$$

undef $r = 0$

not in domain

$$4\pi r^3 - 1600 = 0$$

$$r^3 = \frac{400}{\pi}$$

$$r = \sqrt[3]{\frac{400}{\pi}}$$ check

comment: diam = height

Compact Domain

$[a, b]$

Eg: $f(x) = \frac{x^2}{x-1} = \frac{x^2-1}{x-1} + \frac{1}{x-1}$

$[3, 7]$ $= x+1 + \frac{1}{x-1}$

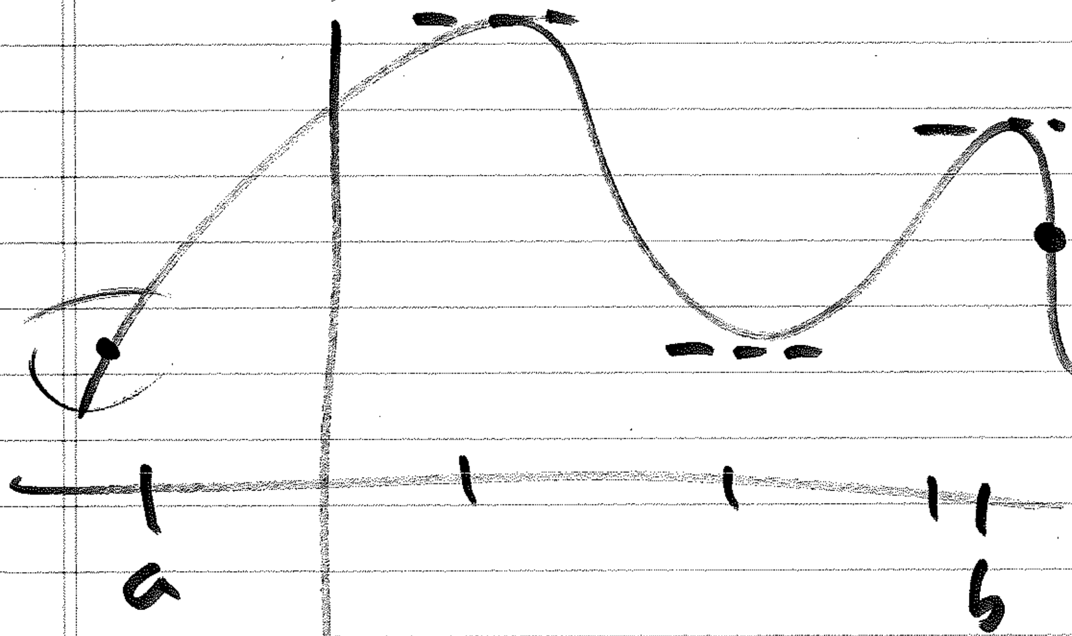
$f'(x) = 1 - \frac{1}{x-1}$

undef $x=1$ not in domain!

$1 - \frac{1}{x-1} = 0$

$1 = \frac{1}{x-1}$ $x-1 = 1$

$x = 2$



x	$f(x)$
CV	x
end pts	x
	x

Check

x	$g(x)$
3	$9/2$ ← min
7	$49/6$ ← max

<u>Thm</u>: ~~Exa~~ If $f(x)$ is cont on $[a, b]$ then f has a max and a min on that interval

Eg: $g(t) = t^2 e^{-t}$

$[-1, 3]$

$$g'(t) = 2t e^{-t} - t^2 e^{-t}$$

$$= (2t - t^2)\underbrace{e^{-t}}_{\neq 0} = 0$$

$$2t - t^2 = 0$$

$$t = 0, 2$$

t	$g(t)$	
-1	$e \approx 2.7$	$\leftarrow$ global max
0	0	$\leftarrow$ min global
2	$4/e^2 \approx 4/9$	
3	$9/e^3 \approx \frac{1}{3}$	