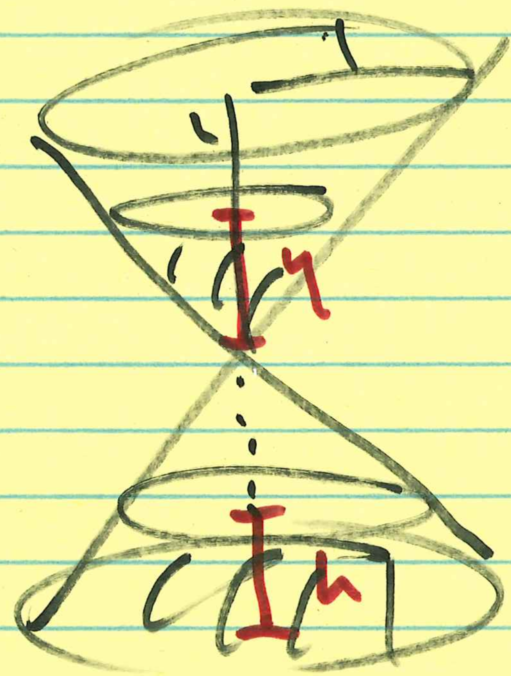
HW 3.9 part II
due Fri

Eg:



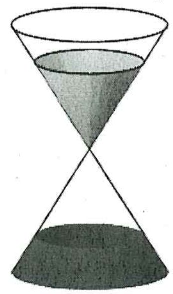
$h=2$
$r=1$

$V = \frac{2\pi}{3}\ cm^3$ total vol

$\frac{dy}{dt} = -1\ cm/sec$

$\frac{dh}{dt} = ?$ when $h=1$

Problem 3. An hourglass is made up of two glass cones connected at their tips (as in the diagram below). Both cones have radius 1 cm and height 2 cm. When the hourglass is flipped over, sand starts falling to the lower cone.

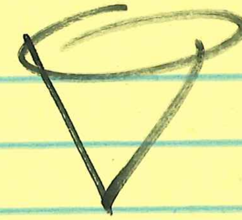


(a) When the sand remaining in the *upper cone* has height y cm, give a formula for its volume A in terms of y.

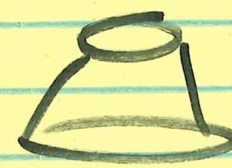
(b) When the sand in the *lower cone* has reached a height of h cm, give a formula for its volume B in terms of h.

(c) Assume that the total volume of sand is $2\pi/3$ cm^3 and that the height of the sand in the upper cone is decreasing at a rate of 1 cm/sec. At the instant that the sand in the lower cone is 1 cm high, determine the rate at which the height of the sand in the lower cone is increasing.

V_T cone

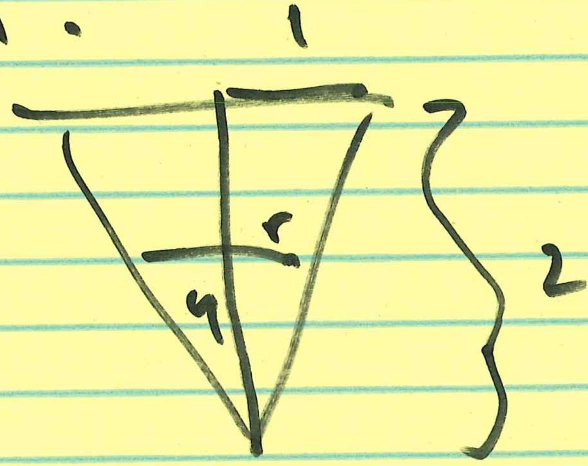


V_B "frustum"



(a) $V_T = \frac{1}{3}\pi r^2 y$

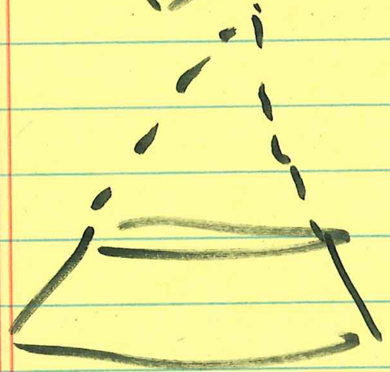
r ?



$\frac{r}{y} = \frac{1}{2}$ $\quad r = \frac{1}{2} y$

$V_T = \frac{1}{3}\pi \left(\frac{1}{2}y\right)^2 y$

(b) V_B



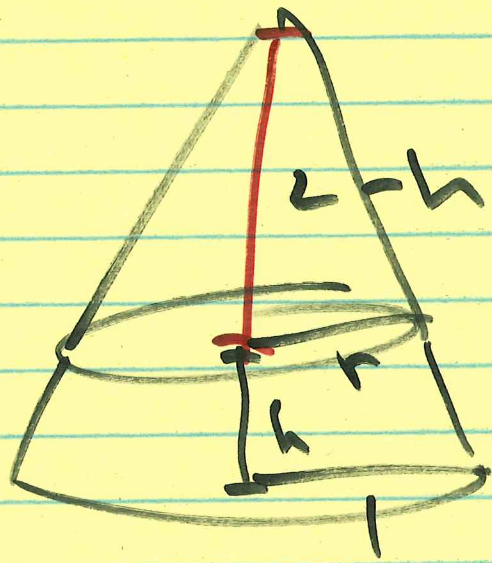
V_{tot} − little cone

$$\frac{1}{3}\pi(1)^2 \cdot 2 - \frac{1}{3}\pi r^2 (2-h)$$

$$\frac{r}{2-h} = \frac{1}{2}$$

$$r = \frac{1}{2}(2-h)$$

$$V_B = \frac{2\pi}{3} - \frac{1}{3}\pi\left[\frac{1}{2}(2-h)\right]^2(2-h)$$



$r = ?$

$V_B \qquad V_T$

$$-\frac{dV_T}{dt} = \frac{dV_B}{dt} \quad \checkmark$$

$$V_B = \frac{2\pi}{3} - \frac{\pi}{12}(2-h)^3$$

Linear Approximation

Eg: linearization

tangent line

$f(x) = \tan^{-1} x$

local - at a point

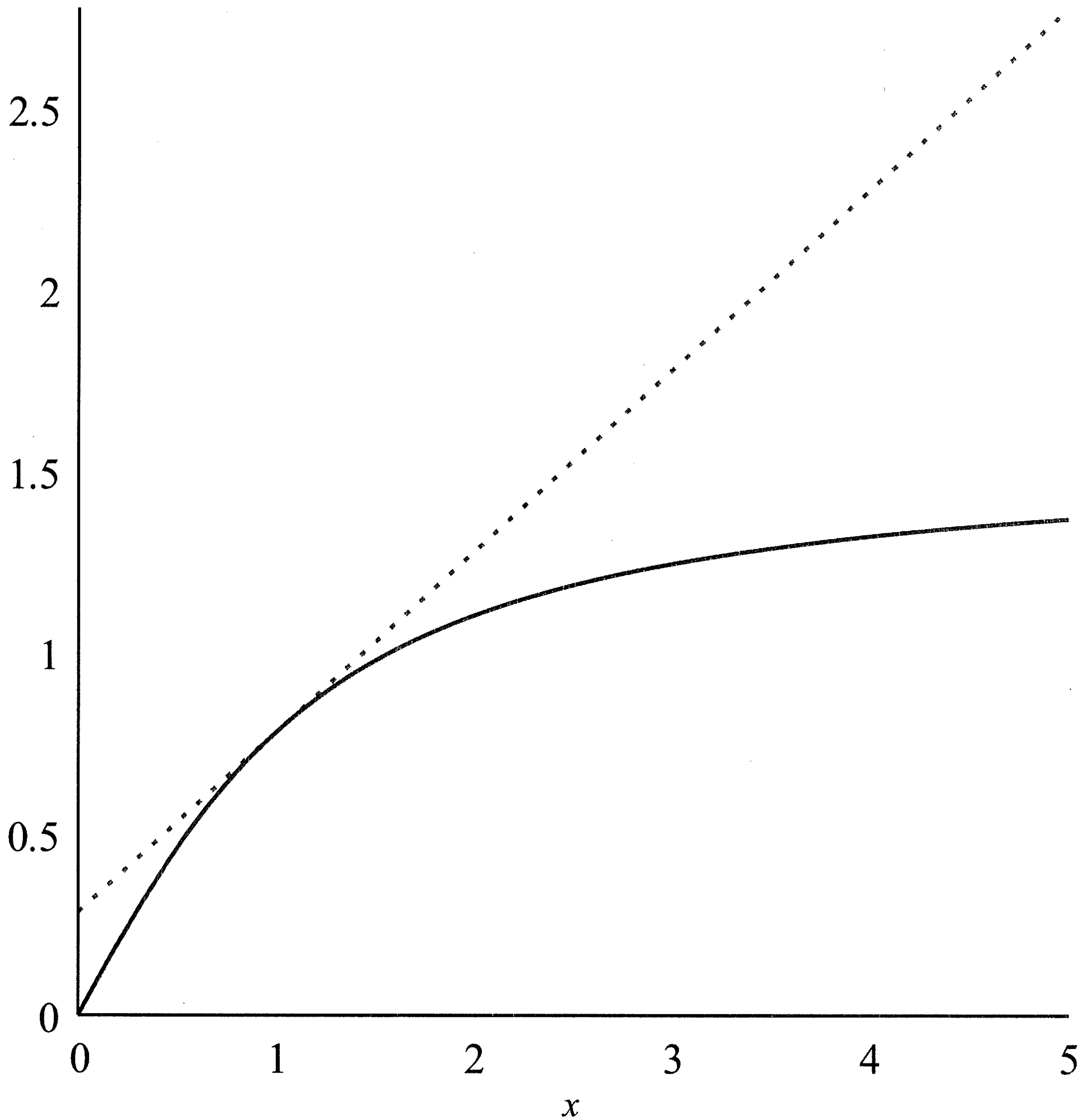
$x = 1$

$x = 1 \qquad f(1) = \pi/4$

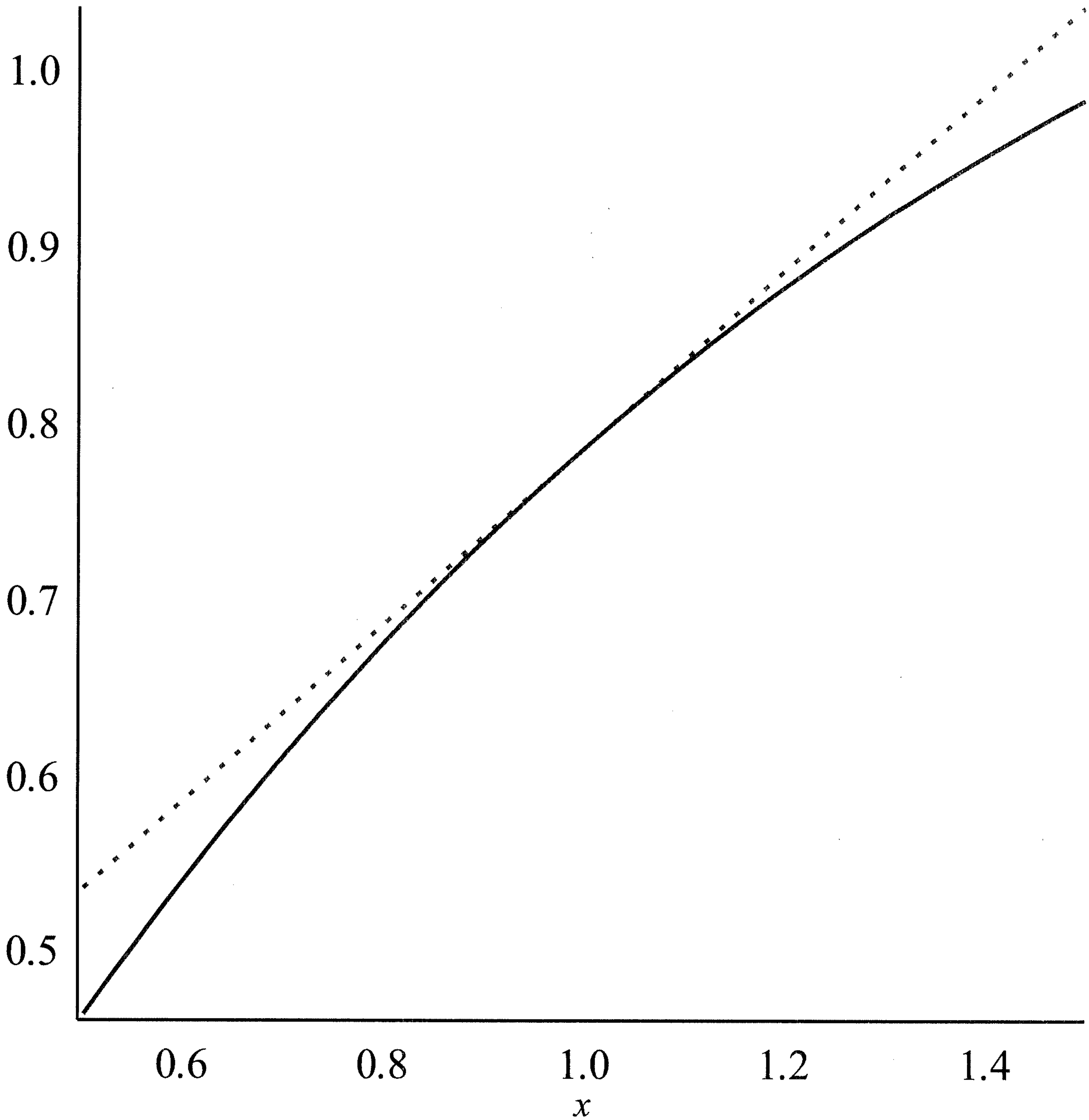
$f'(x) = \frac{1}{1+x^2}$

$f'(1) = \frac{1}{2}$

$y - \pi/4 = 1/2(x-1)$



$y = \tan^{-1} x$ and its linear approximation at $x = 1$



$y = \tan^{-1} x$ and its linear approximation at $x = 1$
Close up

$L_{f,1}(x) = \frac{1}{2}(x-1) + \pi/4$

Thm: $x=a$ f diff

at $x=a$

if $f''(a) < 0$

then tangent is above the curve

$f''(a) > 0$

below

note $L_f(x)$

is pretty good

$[0.8, 1.2]$

<u>note</u>: overestimate

$\tan^{-1}(0.9) \approx L_f(0.9)$

$= \frac{1}{2}(\underbrace{0.9-1}_{-0.1}) + \pi/4$

$= \pi/4 - 0.05$

a little too big

tangent above curve.

$$f(x) = \tan^{-1} x$$

$$f'(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$$

$$f''(x) = -(1+x^2)^{-2} \cdot 2x$$

$$= -\frac{2x}{(1+x^2)^2}$$

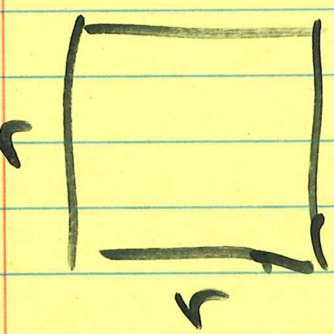
$$f''(1) = -\frac{2}{4} = -\frac{1}{2}$$

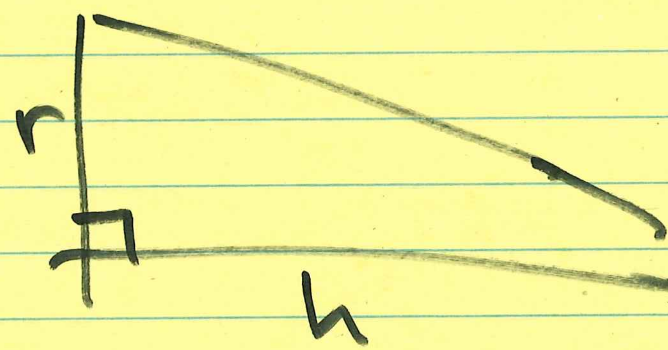
Eg: Story

72 ft fencing

make 2 enclosures

square   right triangle





to begin $r = 9$ $h = 12$

$4 \cdot 9 + r + h + \sqrt{r^2 + h^2}$

$4r$

$9 + 12$

$4 \cdot 9 + 12 + 9 + 15$

$\cancel{36 + 28}$

$36 + 21 + 15 = 72$ ✓

increase h to 12.4

approx r

$4r + r + h + \sqrt{r^2 + h^2} = 72$

$\underset{r}{(9,}\ \underset{h}{12)}$ $\quad \dfrac{dh}{dr}$

$h - 12 = m(r - 9)$

$m = -28/29$

$$\frac{d}{dr}$$

$$0 = 4 + 1 + \frac{dh}{dr} + \frac{1}{2\underbrace{\sqrt{r^2+h^2}}_{15}} \cdot \left(2\underset{9}{r} + 2\underset{12}{h}\frac{dh}{dr}\right)$$

$$0 = 5 + \frac{dh}{dr} + \frac{1}{30}\left(18 + 24\frac{dh}{dr}\right)$$

$$0 = 5 + \frac{dh}{dr} + \frac{3}{5} + \frac{4}{5}\,dh/dr$$

$$-\frac{28}{5} = \frac{29}{5}\,dh/dr$$

$$-\frac{28}{29} = dh/dr$$

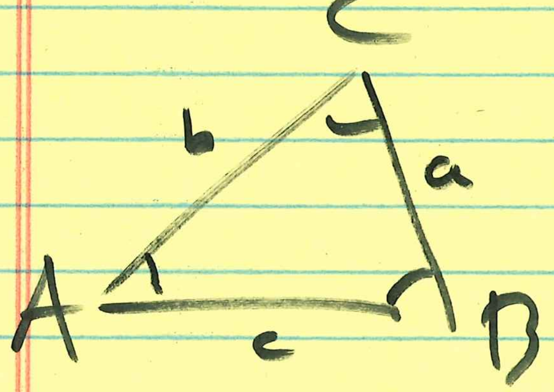
$$\overset{12.4}{h} - 12 \approx -\frac{28}{29}(r-9)$$

$$\underbrace{h-12}_{0.4}$$

$$\Delta h \approx -\frac{28}{29}\,\Delta r$$

$$0.4 \cdot \frac{-28}{29} \approx r-9$$

$$\frac{\Delta h}{\Delta r} \approx \frac{dh}{dr}$$



Law of Sines

$$\boxed{\frac{\sin B}{b} = \frac{\sin A}{a}} = \frac{\sin C}{c}$$

$$\frac{b}{a} = \frac{\sin B}{\sin A}$$

$A = \pi/4 \qquad B = \pi/3$

increase A to 46°

how much does B inc?

$$1\ \mathrm{deg} \cdot \frac{\pi\ \mathrm{rad}}{180\ \mathrm{deg}}$$

$$\Delta A = \frac{\pi}{180}\ \mathrm{rad}$$

$$\frac{b}{a} = ?$$