Log diff

Chain Rule

terminology

- slope of tan
- velocity
- acceleration

$$y' = x^{x+1/x^2+1} \cdot (\quad)$$

param eqn's

$\frac{dx}{dt}$ $\frac{dy}{dt}$ dy/dx

circles / ellipses

projectile

speed $\sqrt{\dot{x}^2+\dot{y}^2}$

implicit diff

(12) 1. Compute the derivative of the following functions. You need not simplify your answer, but your final answers must give the derivative in terms of x.

a. $y = \sqrt{(1 + x^3)^{\frac{1}{3}} + 1}$

b. $y = x^{\frac{x+1}{x^2+1}}$

$$y = \sqrt{(1+x^3)^{1/3}+1}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{(x^3+1)^{1/3}+1}} \cdot \frac{1}{3}(1+x^3)^{-2/3} \cdot 3x^2$$

$$y = x^{\frac{x+1}{x^2+1}}$$

$$\ln y = \ln x^{\frac{x+1}{x^2+1}}$$

$$= \left(\frac{x+1}{x^2+1}\right) \cdot \ln x$$

$$\frac{y'}{y} = \frac{1(x^2+1) - 2x(x+1)}{(x^2+1)^2} \ln x + \frac{x+1}{x^2+1} \cdot \frac{1}{x}$$

Problem 4. Beginning at time $t = 0$ seconds, an ant crawls according to the equations

$$x(t) = t^3 + 45t + 1 \qquad \text{and} \qquad y(t) = -12t^2.$$

(a) At what times t within the first 10 seconds is the ant's direction of travel parallel to the line $x + y = 2$? (*Please round your answer to three digits after the decimal.*)

(b) At what time within the first 10 seconds does the ant attain its maximal speed? (*Please round your answer to three digits after the decimal.*)

$x = t^3 + 45t + 1$

$y = -12t^2 \qquad 0 \le t \le 10$

parallel to $x + y = 2$

$\frac{dy}{dx} = -1 \qquad y = -x + 2$

$\dot{x} = 3t^2 + 45$

$\dot{y} = -24t$

$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{-24t}{3t^2+45} = -1$

$$+24t = +1 \cdot (3t^2 + 45)$$

$$24t = 3t^2 + 45$$

$$3t^2 - 24t + 45 = 0$$

$$t^2 - 8t + 15 = 0$$

$$(t-3)(t-5) = 0$$

calc speed

$$s = \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$= \sqrt{(3t^2+45)^2 + (\underset{-24t}{\cancel{-12t}})^2}$$

3. The point $(2, 0)$ is on the curve given by the equation

$$3xe^y - x^2y + 4y = 6.$$

(a) (10 points) Use linear approximation to estimate the value of c if $(c, 0.015)$ is also on the same curve.

(b) (4 points) Compute the value of y'' when $x = 2$ and $y = 0$ and use it to determine if the estimate for c you found in part (a) is more than or less than the actual value of c.

$3xe^y - x^2y + 4y = 6$

$(2,0)$ $x=2$

$y=0$

y'

y''

~~3xe~~

$3e^y + 3xe^y \cdot y' - \cancel{2xy} - x^2y'$

$+4y' = 0$

$3 \cdot e^0 + 3 \cdot 2 \cdot e^0 \cdot y' - 0 - \cancel{4y'} + \cancel{4y'} = 0$

$3 + 6y' = 0$ $y' = -1/2$

$3e^y y' + \cancel{3e^y y'} + \cancel{Bxx}$

$+ 3 \cdot 1 e^y y' + 3xe^y y' y'$

$+ 3xe^y y''$

$-2y - 2xy' - 2xy' - x^2 y''$

$+4y'' = 0$

$x = \cancel{0}\ 3$

$y = 0 \quad e^0 = 1$

$3\cancel{y'} + 3\cancel{y'} + 6y'y' + 6y''$

$-0 - \cancel{4y'} - 4y' - \cancel{4y''}$

$+\cancel{4y''} = 0$

3. (10 pts) At time t, the location of a particle is given by:

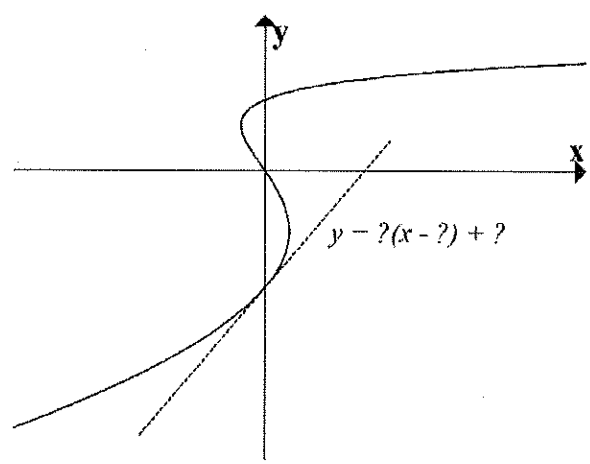
$$x = t^3 - t \quad , \quad y = 8te^{(-t/4)}$$

(a) Find all times t when the curve has a horizontal or vertical tangent.

i. Horizontal Tangent time(s):

ii. Vertical Tangent time(s):

(b) Find the equation for the tangent line at the negative y-intercept (shown in picture). Leave your numbers in exact form.



$x = t^2 - t$

$y = 8t \cdot e^{-t/4}$

✓ horiz $dy/dx = 0$ $\frac{\dot{y}}{\dot{x}} = 0$ $\dot{y} = 0$

vert

$\dot{x} = 0$

$\dot{y} = 8e^{-t/4} + 8t \cdot e^{-t/4} \cdot -1/4 = 0$

$8e^{-t/4} - 2te^{-t/4} = 0$

$2e^{-t/4}(4 - t) = 0$