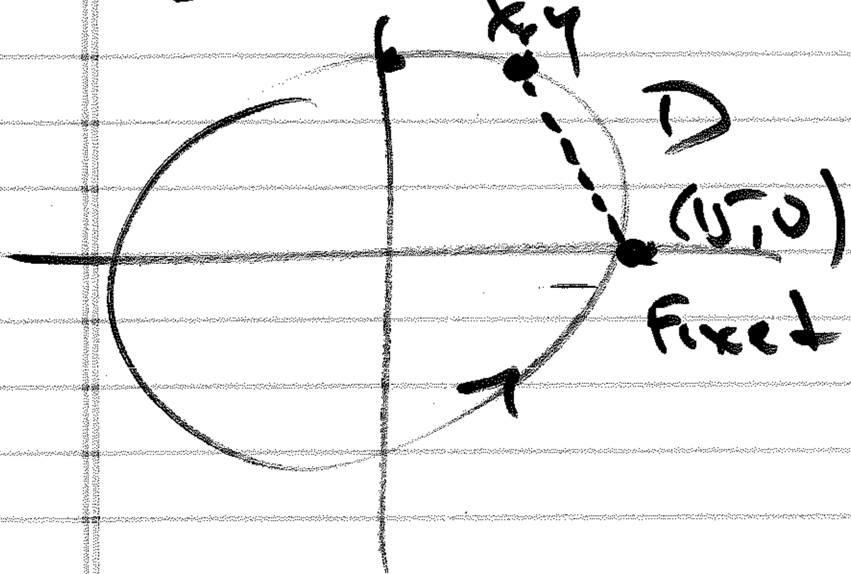
Midterm II
Tuesday - 3.6
10.2
10.1

Chain Rule
- implicit
- param eqn's
- log

no need
- related rates
story probl
- linearization
- max/min - optimization

no limits

Related Rates



$$\frac{dD}{dt} = ?$$

diam = 30 $\qquad r = 15$ ft

1 rev in 2 min

$$\frac{1\ \cancel{\text{rev}}}{2\ \text{min}} \cdot \frac{2\pi\ \text{rad}}{\cancel{\text{rev}}} \qquad \omega = \pi\ \frac{\text{rad}}{\text{min}}$$

$$\theta_0 = 0$$

$$2 \cdot 15\sqrt{2} \cdot \frac{dD}{dt}$$

$$= 2(0+15)(-15\pi) + 0$$

$$= -2 \cdot 15^2 \pi$$

$$\frac{dD}{dt} = -\frac{15\pi}{\sqrt{2}} \text{ ft/sec}$$

OR

$$D = \sqrt{(15\cos\pi t - 15)^2 + (15\sin\pi t)^2}$$

$$\frac{dD}{dt} = \ldots$$

① $x = 15\cos \pi t$

$y = 15 \sin \pi t$

$t = \frac{1}{2}$ min

② $D^2 = (x-15)^2 + y^2$ $\quad t = 30$ sec

$2 D \frac{dD}{dt} = 2(x-15)\boxed{\frac{dx}{dt}} + 2y\boxed{\frac{dy}{dt}}$

0

$D = \sqrt{(0-15)^2 + 15^2} = 15\sqrt{2}$

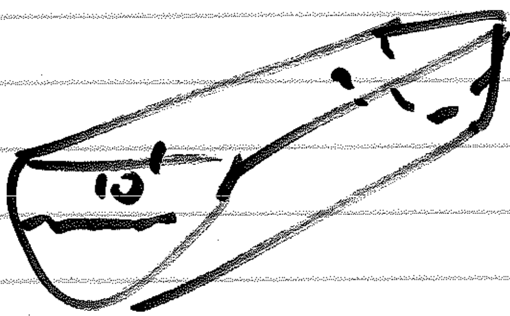
$x = 15 \cos \pi/2 = 0$

$y = 15 \sin \pi/2 = 15$

$\frac{dx}{dt} = -15\pi \sin \pi t \Big|_{t=1/2} = -15\pi$

$\frac{dy}{dt} = 15\pi \cos \pi t \Big|_{t=1/2} = 0$ ✓

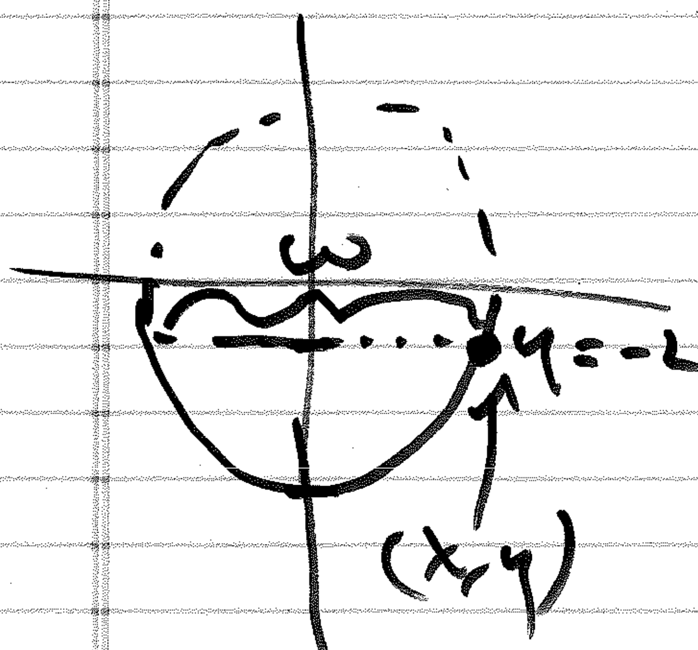
Eg: Trough



Semi circular
X-section

~~dua~~ depth increasing
6 in/sec

at what rate is
the width (top of water)
increasing
8 ft ~~deep~~?



$r=5$

$x^2+y^2=25$

$x^2+(-2)^2=25$

$x^2=21$

$\frac{dy}{dt}=\frac{1}{2}$ ft/sec

$w=2x$ $\qquad$ $\frac{dw}{dt}=2\frac{dx}{dt}$

$\frac{dx}{dt}=?$

$$\frac{d}{dt}\, x^2 + y^2 = \frac{d}{dt}\, 25$$

$$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = 0$$

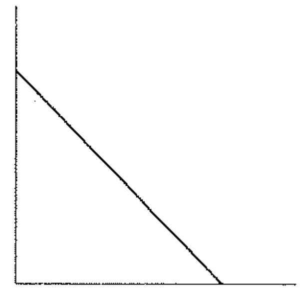
$$\sqrt{21} \quad ? \quad -2 \cdot \frac{1}{2}$$

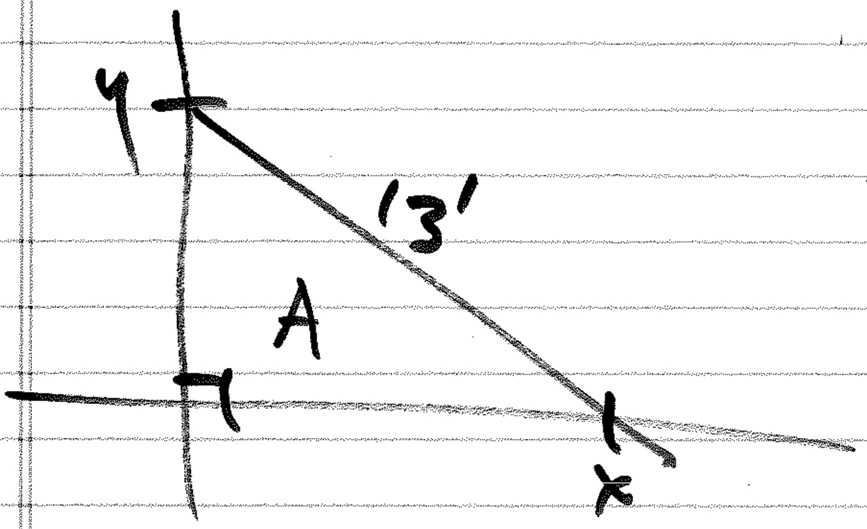
$$\sqrt{21}\, \frac{dx}{dt} = 1$$

$$\frac{dx}{dt} = \frac{1}{\sqrt{4}} \text{ ft/sec}$$

$$\frac{dw}{dt} = 2 \cdot \frac{1}{\sqrt{21}} = \frac{2}{\sqrt{21}} \text{ ft/sec}$$

$\boxed{3}$ (10 points) A ladder 13 ft long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 0.5 ft/s. Consider the triangle formed by the ladder, the ground and the wall. At what rate is the area of this triangle changing when the bottom of the ladder is 12 ft from the wall?





$$\frac{dx}{dt} = \frac{1}{2}\ \text{ft/sec}$$

$$\frac{dA}{dt} = ? \quad \text{when } x = 12$$

$$A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{13^2 - x^2}$$

$$5^2 + 12^2 = 13^2$$

(1) $\frac{dA}{dt} = \frac{1}{2} \underbrace{\frac{dx}{dt}}_{t} \cdot \underbrace{y}_{5} + \frac{1}{2} \underbrace{x}_{12} \underbrace{\frac{dy}{dt}}_{\boxed{-6/5}}$

(2) $x^2 + y^2 = 13^2$

$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = 0$

$12 \cdot \frac{1}{2} + 5 \cdot \square$

$6 + 5 \frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{6}{5}$

$\frac{dA}{dt} = \frac{1}{2} \cdot \frac{1}{2} \cdot 5 + \frac{1}{2} \cdot 12 \cdot \frac{-6}{5}$ ft^2/sec