Logarithms

Eg: $y = x^{\tan x}$

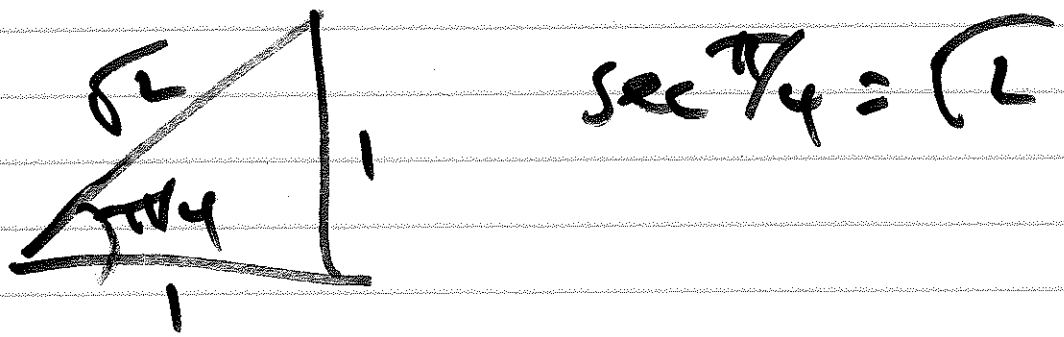
write tan line at $x = \pi/4$

$$y(\pi/4) = (\pi/4)^{\tan \pi/4}$$

$$= (\pi/4)^1 = \pi/4$$

$$y - b = m(x - a)$$

$$y - \pi/4 = m(x - \pi/4)$$



$\sec \pi/4 = \sqrt{2}$

$$\ln y = \ln x^{\tan x}$$

$$y' = \tan x \cdot \ln x$$

$$\frac{y'}{y} =$$

$$\ln$$

$$\frac{y'}{y} = \sec^2 x \cdot \ln x + \tan x \cdot \frac{1}{x}$$

$$m = y' \quad x = y = \pi/4$$

$$\frac{y'}{\pi/4} = 2 \cdot \ln \pi/4 + 1 \cdot \frac{1}{\pi/4}$$

$$=$$

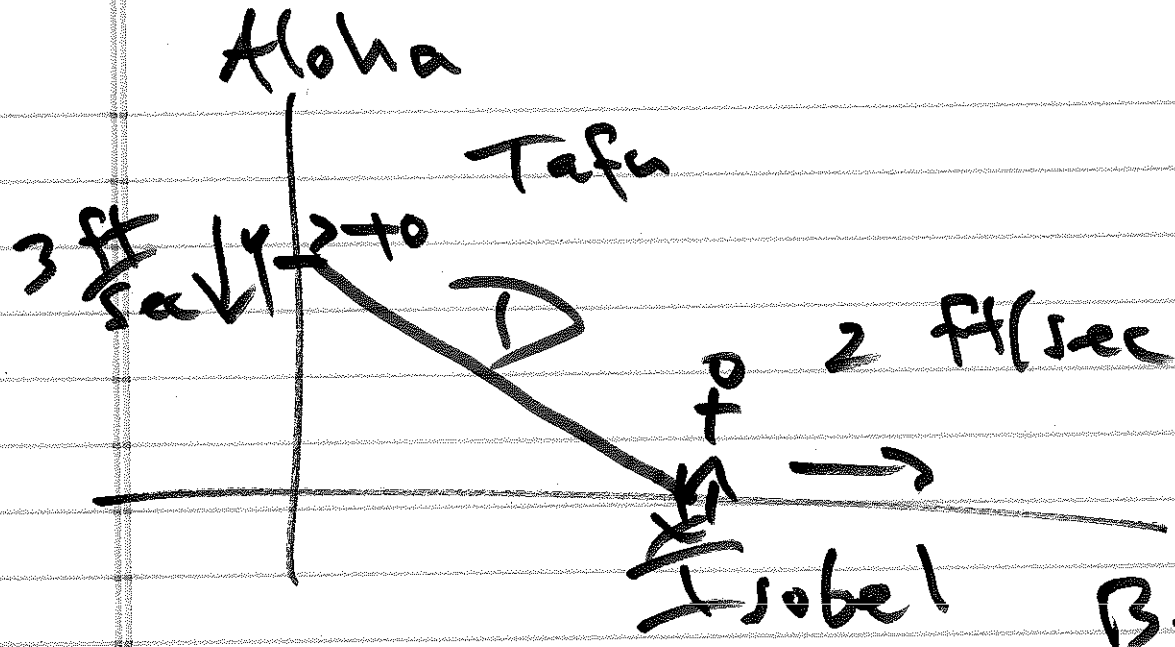
$$y' = \pi/4 \left(2 \ln \pi/4 + 4/\pi \right)$$

$$m = \left(\frac{\Sigma}{\Sigma} \ln \pi/4\right) + 1$$

$$y - \frac{\Pi}{4} = \left(1 + \frac{\Sigma}{\Sigma} \ln \pi/4\right)\left(x - \pi/4\right)$$

Worksheets - Geometry

Related Rates



$t = 0$ I at intersection

T 30 ft N

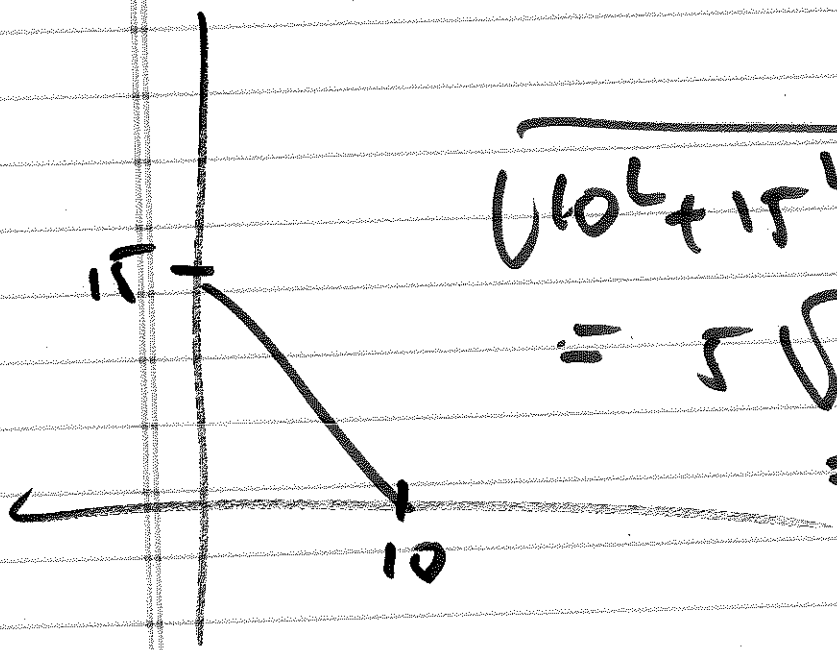
D distance between

how is it changing

when $t = ?$

$D = \sqrt{x^2 + y^2}$

$D^2 = x^2 + y^2$



$\sqrt{10^2+15^2}$

$= 5\sqrt{2^2+3^2}$

$= 5\sqrt{13}$

$t=5$

$\frac{d}{dt}x^2 = \frac{d}{dt}x^2+y^2$

$2x\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$10.2 \quad -3.15$

$\frac{dy}{dt} = -3$

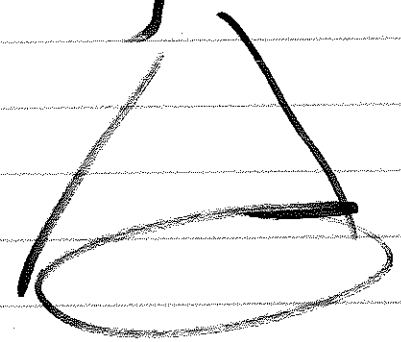
$$5\sqrt{13}\,\frac{dD}{dt} = -25$$

$$\frac{dD}{dt} = -\frac{5}{\sqrt{13}}\ \text{ft/sec}$$

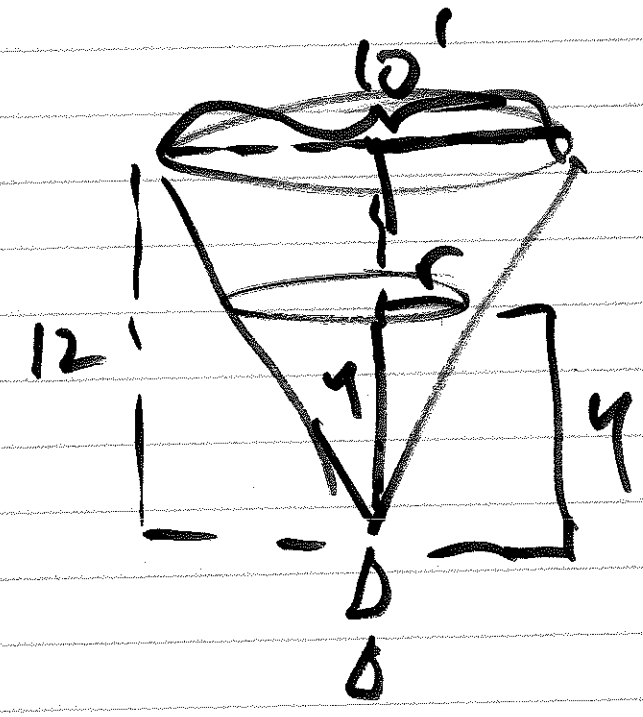
Pythag

similar triangles

geometry



Ex:



$$\frac{dV}{dt} = -2 \text{ ft}^3/\text{sec}$$

$$y = 6$$

$$\frac{dy}{dt} = ?$$

$$V = \frac{1}{3}\pi r^2 h$$

$h = 4$ $\quad r = ?$

$$\frac{r}{4} = \frac{5}{12}$$

$$r = \frac{5}{12}4$$

constraint

$$V = \frac{1}{3}\pi\left(\frac{5}{12}4\right)^2 \cdot 4$$

$$V = \frac{1}{3}\cdot\frac{25}{144}\pi^4 4^3$$

$$\frac{dV}{dt} = \frac{25}{432}\pi \left[3y^2 \frac{dy}{dt} \right]$$

$y = 6$

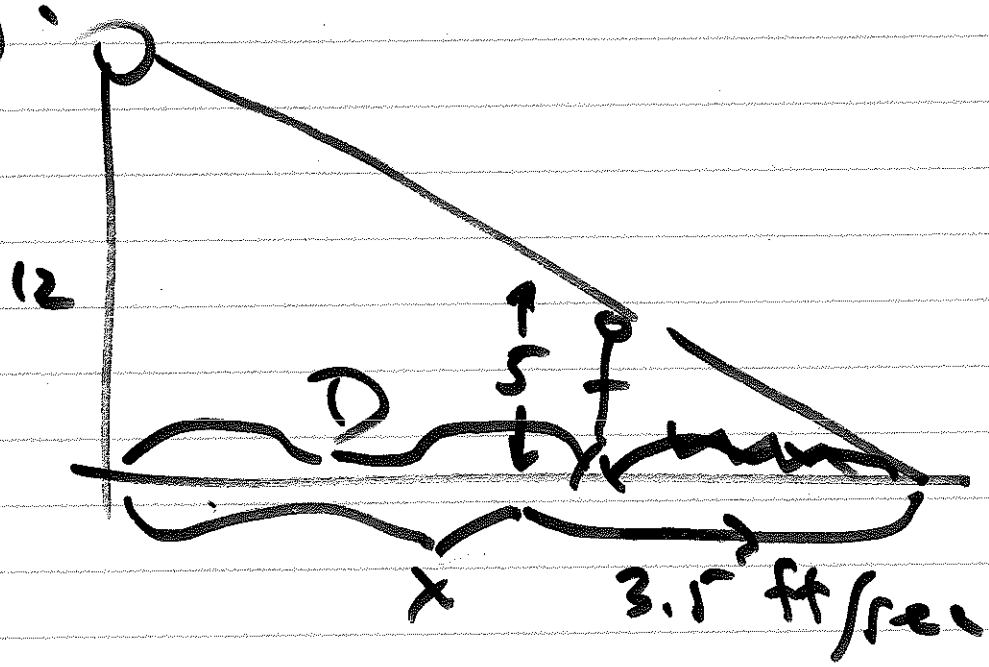
-2

$$-2 = \frac{25\pi}{432} \cdot \frac{25\pi}{144} \cdot 36 \frac{dy}{dt}$$

$$-2 = \frac{\pi}{2} \, dy/dt$$

$$\frac{-4}{\pi} \text{ (ft/sec)} = dy/dt$$

Ex:



how fast is length of shadow changing

how fast is the tip of her shadow moving?

$$\frac{x}{12} = \frac{x-D}{5} \qquad \frac{dD}{dt} = 3.5$$

$$= \frac{x}{5} - \frac{D}{5}$$

$$\frac{D}{5} = \frac{x}{5} - \frac{x}{12}$$

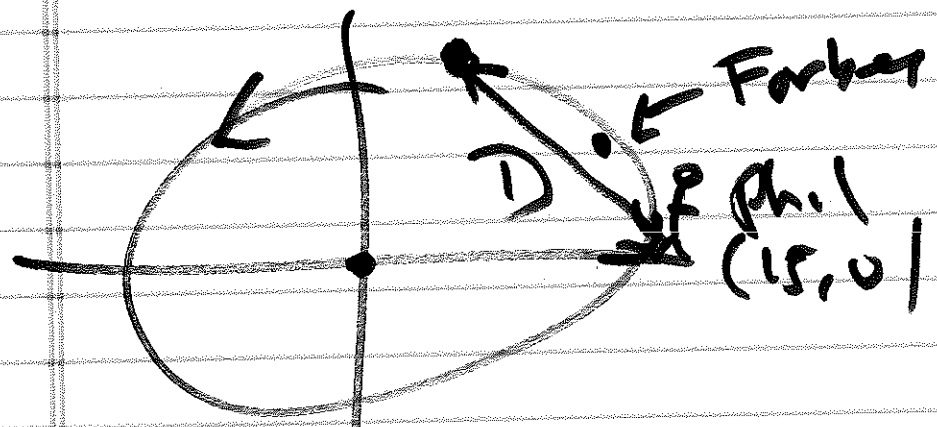
$$\frac{D}{5} = \frac{7x}{60} \leftarrow$$

$$\frac{dD/dt}{5} = \frac{7}{60} \cdot \frac{dx}{dt}$$

$$\frac{3.5}{5} = \frac{7}{60} \cdot dx/dt$$

$$\frac{ft}{sec} \quad \frac{3.5}{5} \cdot \frac{60}{7} = dx/dt$$

Ex: Merry-go-round



Diam = 30 ft r = 15

1 rev every 2 min

$t = 30$ sec $\frac{1}{2}$ min

$\frac{dD}{dt} = ?$

D = dist from $(15, 0)$ to (x, y)

$$D^2 = (x-15)^2 + y^2$$

① $$2D\frac{dD}{dt} = 2(x-15)\cdot\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$D = ?$ $\sqrt{(0-15)^2 + (15-0)^2}$

$x = ?$ 0

$y = ?$ 15

② $\frac{dx}{dt}$ $t = 1/2$

$\frac{dy}{dt}$

parametric equations

$x = 15 \cos\theta$

$y = 15 \sin\theta$

$\theta = \omega t + \theta_0$

$\theta_0 = 0$

$\omega = \frac{1 \text{ rev}}{2 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \pi \frac{\text{rad}}{\text{min}}$

$$\begin{cases} x = 15 \cos \pi t \\ y = 15 \sin \pi t \end{cases} \quad t = 1$$