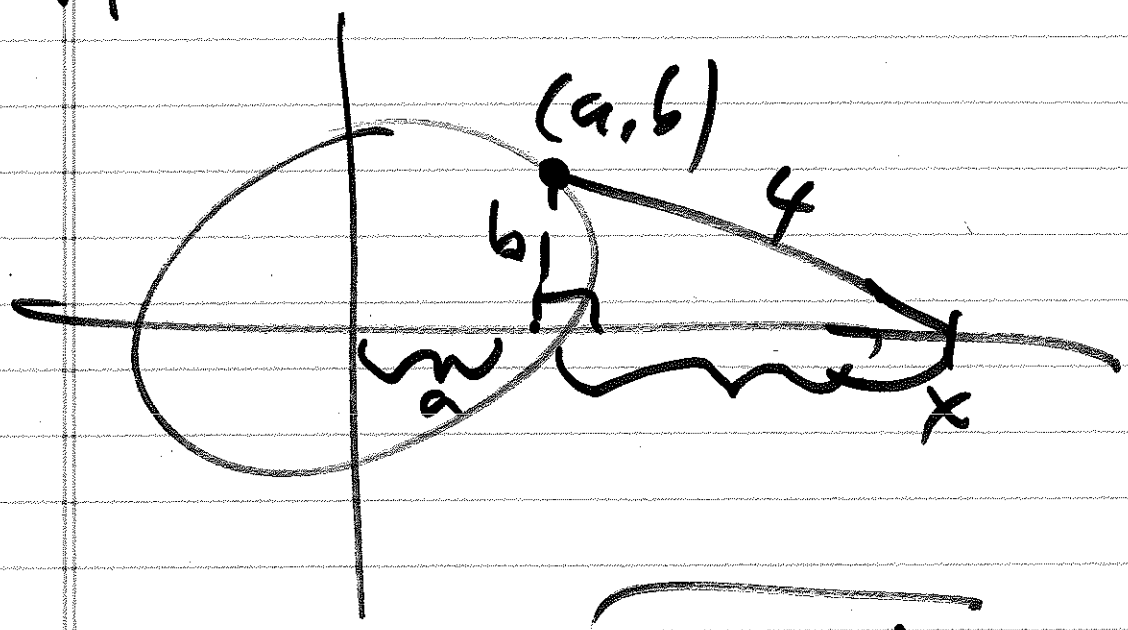
HW



$$x(t) = a + \sqrt{16 - b^2}$$

$r = 2$

$a = 2\cos\theta$ $\qquad \theta = \omega t + \theta_0$

$b = 2\sin\theta$

ω $\frac{\text{rad}}{\text{sec}}$

(c) speed = |velocity|

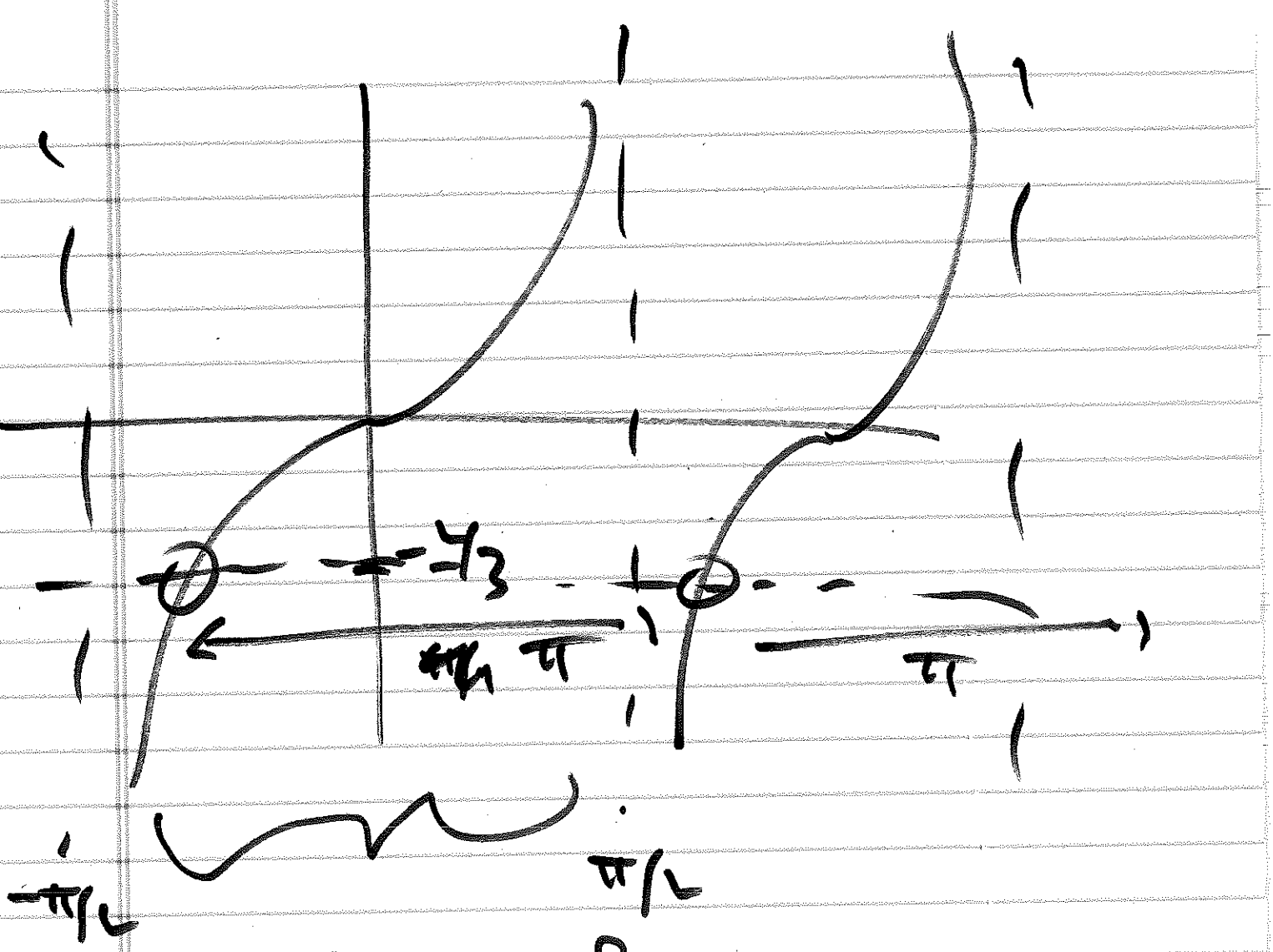
Midterm II
- Logarithms

Ex: $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$y = a \sin\theta$ $\quad\quad$ $\theta = t$

$x = b \cos\theta$

$$\begin{cases} y = 2 \sin t \\ x = 3 \cos t \end{cases}$$


-π/2
π/2
principal domain

- when is horiz vel equal to vertical vel?

$$V_x = \frac{dx}{dt} = -3\sin t$$

$$V_y = \frac{dy}{dt} = 2\cos t$$

$$-3\sin t = 2\cos t$$

$$\frac{\sin t}{\cos t} = -\frac{2}{3}$$

$$\tan t = -2/3$$

$$t = \tan^{-1}(-2/3) + n\pi$$ ✓

Eqn of tan line?

a. $x = 3\cos t$

$= 3\cos\tan^{-1}(-4/3)$

b. $y = 2\sin t$

$= 2\sin\tan^{-1}(-4/3)$

$y - b = m(x - a)$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 1$

Logs $\log_b x$

inverse b^x

normally we $b = e$

$\ln x$ $\log_e x$

$\frac{d}{dx} \ln x = \frac{1}{x}$

1) $\ln AB = \ln A + \ln B$

2) $\ln A/B = \ln A - \ln B$

3) $\ln A^B = B \ln A$

Eg: $y = \ln(x^2+2x)$

$$\frac{dy}{dx} = \frac{1}{x^2+2x} \cdot 2x+2$$

$$= \frac{2x+2}{x^2+2x}$$

Eg: $y = \ln\left(\frac{x+1}{x+3}\right)$

$$= \ln(x+1) - \ln(x+3)$$

$$\frac{dy}{dx} = \frac{1}{x+1} - \frac{1}{x+3}$$

Eg: $y = \dfrac{(2x+3)^9}{x^7\sqrt{3x+5}}$

$\ln y = \ln \dfrac{(2x+3)^9}{x^7\sqrt{3x+5}}$

$= \ln(2x+3)^9 - \ln x^7 - \ln\sqrt{3x+5}$

$= 9\ln(2x+3) - 7\ln x - \frac{1}{2}\ln(3x+5)$

diff both sides

$\dfrac{y'}{y} = 9\cdot\dfrac{2}{2x+3} - 7\cdot\dfrac{1}{x} - \dfrac{1}{2}\dfrac{3}{3x+5}$

$y' = y\left(\quad\quad\quad\quad\right)$

Eg: $y = x^x$

$$\ln(2x+3)^9 - \ln x^7 \sqrt{3x+5}$$

$$= -\left(\ln x^7 + \ln\sqrt{3x+5}\right)$$

$$\ln y = \ln x^{x} \quad (*)$$

$$= x \cdot \ln x$$

$$\frac{y'}{y} = 1 \cdot \ln x + x \cdot \frac{1}{x}$$

$$= 1 + \ln x$$

$$y' = y(1 + \ln x)$$

$$y' = x^x \cdot (1 + \ln x)$$

Eg: $y = (\sec x)^{\ln x}$

$$\ln y = \ln \sec x^{\ln x}$$

$$= \ln x \cdot \ln \sec x$$

$$\frac{y'}{y} = \frac{1}{x} \cdot \ln \sec x + \ln x \cdot \frac{\sec x \tan x}{\sec x}$$

$$= \frac{\ln(\sec x)}{x} + (\ln x)(\tan x)$$

$$y' = y\left(\sim\sim\sim \right)$$

$$y' = (\sec x)^{\ln x}\left(\frac{\ln \sec x}{x} + \ln x \cdot \tan x\right)$$

$$f(x) = x^{x} \quad ?$$

$$f(x) = x^{\pi}$$

$$y = x^{\pi}$$

$$\ln y = \ln x^{\pi}$$

$$= \pi \ln x$$

$$\frac{y'}{y} = \pi \cdot \frac{1}{x}$$

$$y' = y \cdot \frac{\pi}{x}$$

$$= x^{\pi} \cdot \frac{\pi}{x}$$

$$= \pi x^{\pi - 1}$$

Eg: other logs

$\ln \log_{10} x$

$y = \log_{10} x$?

$\log_{10} x$ inverse of 10^x

$10^y = 10^{\log_{10} x} = x$

$\ln 10^y = \ln x$ (y)

$y \cdot \ln 10 = \ln x$

$y = \frac{\ln x}{\ln 10}$ Q.E.D.

$$\frac{d}{dx}\log_{10} x = \frac{d}{dx}\frac{\ln x}{\ln 10}$$

$$= \frac{1}{\ln 10}\cdot\frac{d}{dx}\ln x$$

$$= \frac{1}{x}\cdot\frac{1}{\ln 10}$$