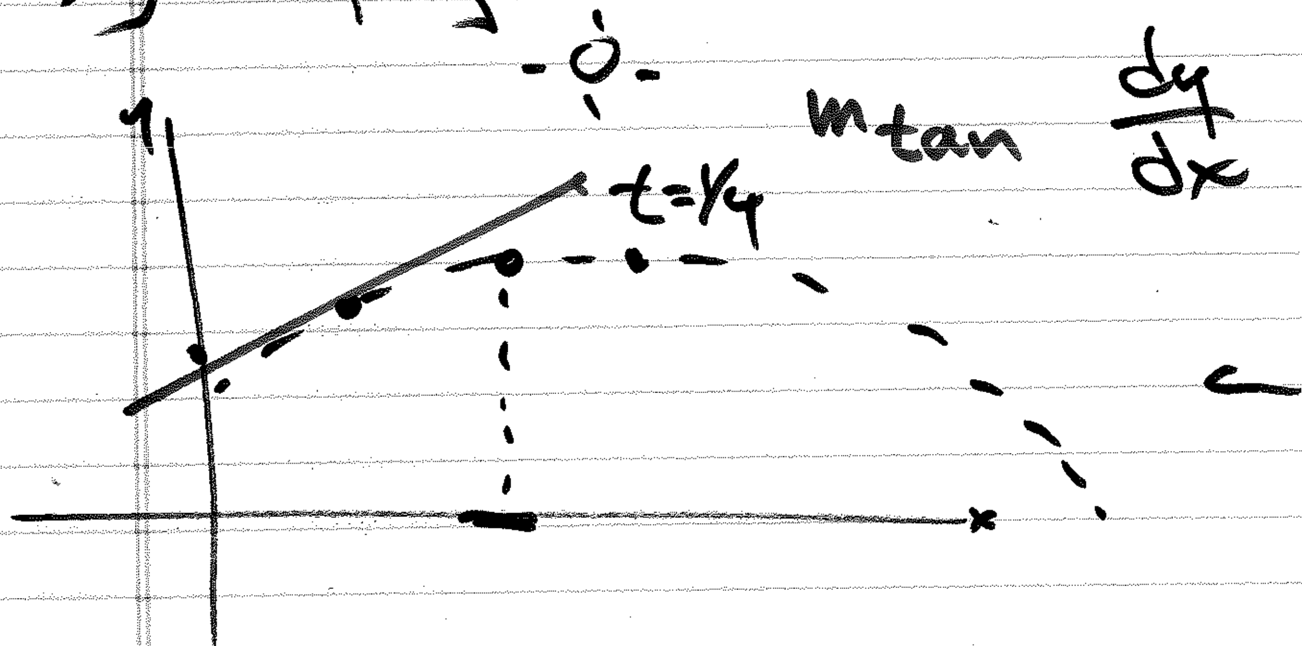
Parametric Equations

Eg: Projectile



$$\begin{cases} y = -16t^2 + 8t + 5 \\ x = 4t \end{cases}$$

Trajectory $y = f(x)$

"Cartesian equation"

$$t = \frac{x}{4}$$

$$y = -16\left(\frac{x}{4}\right)^2 + 8\left(\frac{x}{4}\right) + 5$$

$$y = -x^2 + 2x + 5$$

$$\cancel{4}$$

$$\frac{dy}{dt} = V_y = \dot{y} = -32t + 8$$

$$\frac{dx}{dt} = V_x = \dot{x} = 4 \text{ ft/sec}$$

$$\boxed{\frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{dy}{dt}}$$

chain rule

$y = f(x)$ $\qquad$ $x = g(t)$

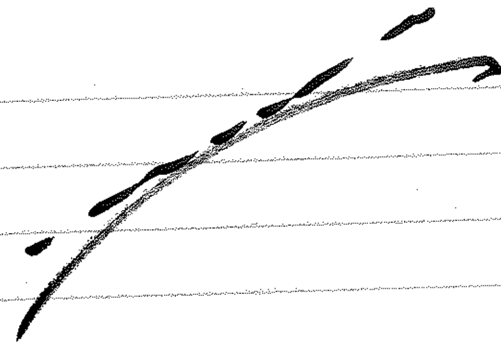
$\frac{dx}{dt} = g'(t)$

$y = f(g(t))$

$$\frac{dy}{dt} = f'(g(t)) \cdot g'(t)$$

$$= f'(x) \cdot \frac{dx}{dt}$$

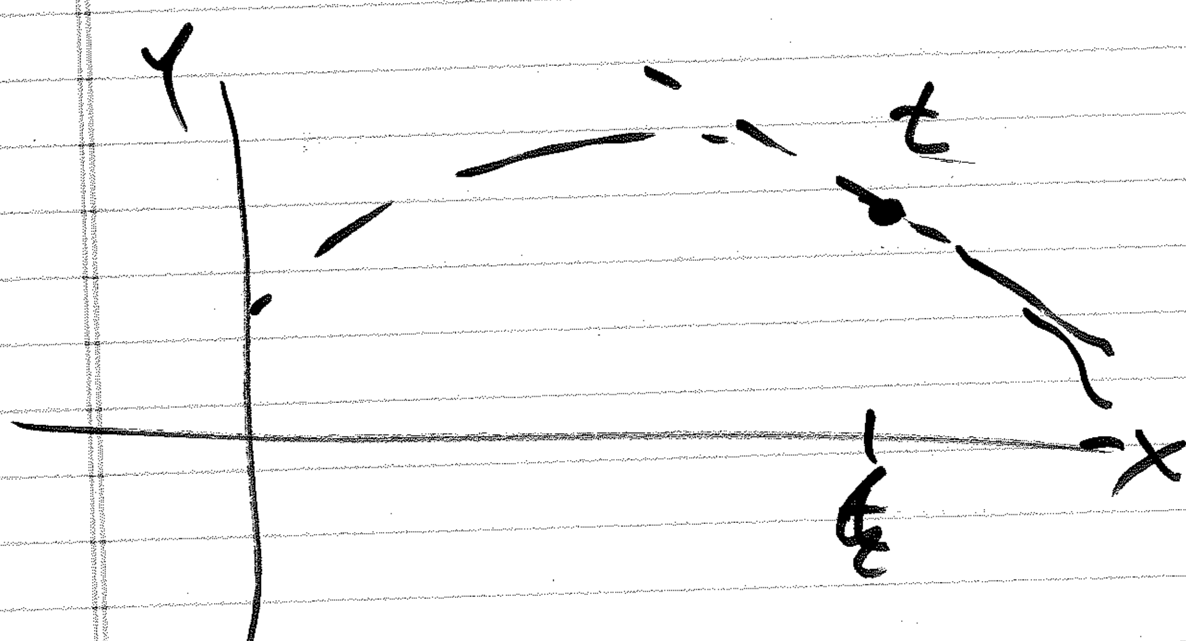
$$\frac{dy}{dx} = m_{tan}$$



$$\frac{dy}{dx} \cdot 4 = -32t + 8$$

$$\frac{dy}{dx} = -8t + 2 = y'$$

$m_{tan}(t)$



$$y' = \frac{dy}{dx}$$

$$y'' = \frac{dy'}{dx} = \frac{d^2y}{dx^2}$$

$$\boxed{\frac{dy'}{dx} \cdot \frac{dx}{dt} = \boxed{\frac{dy'}{dt}}}$$

$$\frac{d^2y}{dx^2} \cdot 4 = \boxed{-8}$$

$$\frac{d^2y}{dx^2} = -2$$

$\boxed{\text{concave up?}}$

y'' + cu — cd

Eg: $\begin{cases} y = 2\sin t \\ x = 5\cos t \end{cases}$ ✓

eliminate parameter

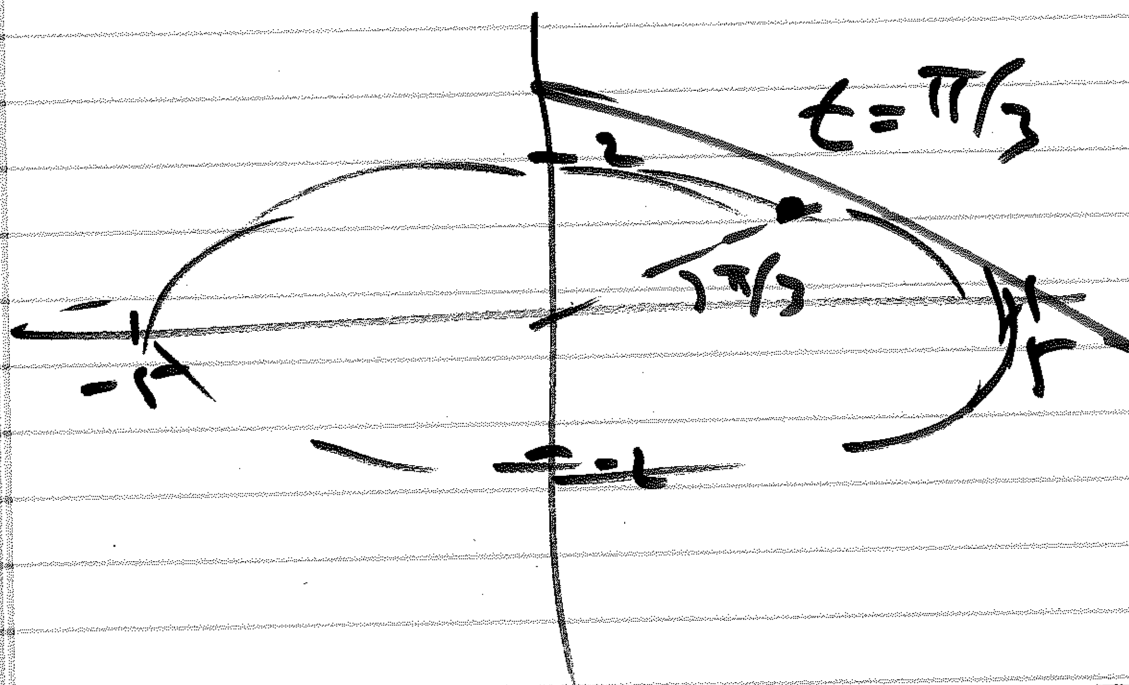
$\frac{y}{2} = \sin t$

$\frac{x}{5} = \cos t$

$\left(\frac{y}{2}\right)^2 + \left(\frac{x}{5}\right)^2 = \sin^2 t + \cos^2 t$

$= 1$

$\frac{y^2}{4} + \frac{x^2}{25} = 1$



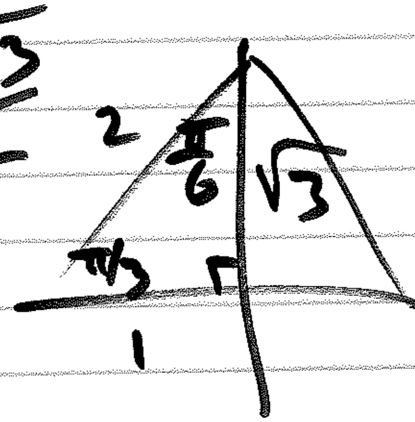
① tan line at $t = \pi/3$

$y - b = m(x - a)$

$a = 5 \cos \pi/3 = 5/2$

$b = 2 \sin \pi/3 = \frac{2\sqrt{3}}{2}$

$= \sqrt{3}$



$$m = \frac{dy}{dx}\Big|_{t=\pi/3}$$

$$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dt} = 2\cos t \qquad t = \pi/3$$

$$2 \cdot 1/2 = 1$$

$$\frac{d\cancel{x}}{dt} = -5\sin t$$

$$-5 \cdot \sqrt{3}/2$$

$$-\frac{5\sqrt{3}}{2}$$

$$= \frac{1}{-\frac{5\sqrt{3}}{2}} \; \frac{2}{2}$$

$$m = -\frac{2}{5\sqrt{3}}$$

$$\cancel{y - \frac{2\sqrt{3}}{2}}$$

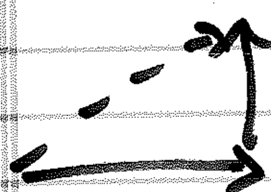
$$y - \sqrt{3} = -\frac{2}{5\sqrt{3}}\left(x - 5/2\right)$$

$$\cancel{4}$$

speed

$$s = \sqrt{\dot{x}^2 + \dot{y}^2}$$

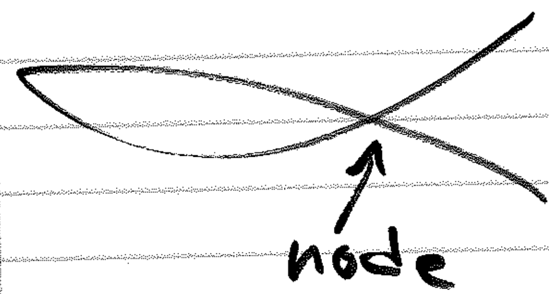
$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

 vectors ... 126

at $t = \pi/3$

$$s = \sqrt{\left(-\frac{5\sqrt{3}}{2}\right)^2 + 1^2} \quad \text{ft/sec}$$

Ex: $\begin{cases} x = t^3 - 4t \\ y = t^2 - 2 \end{cases}$



* x-int $\quad y = 0$
* y-int

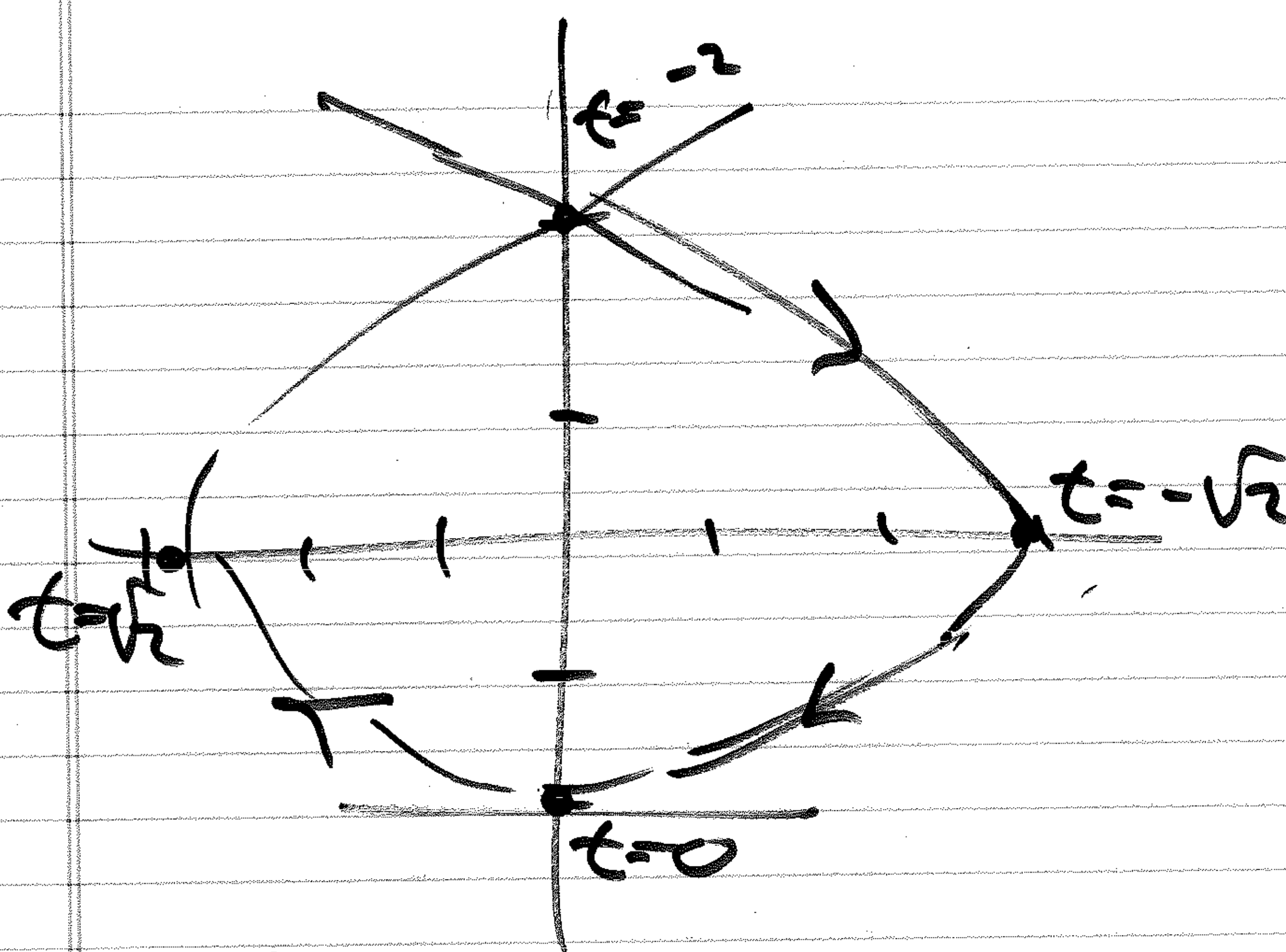
$$t^2 - 2 = 0$$
$$t = \pm\sqrt{2}$$

$t = +\sqrt{2}$

$$x = (\sqrt{2})^3 - 4\sqrt{2}$$
$$= 2\sqrt{2} - 4\sqrt{2}$$
$$= -2\sqrt{2}$$

$t = -\sqrt{2}$

$x = 2\sqrt{2}$



Y-int $\quad x=0$

$$t^3-4t=0$$

$$t(t-2)(t+2)=0$$

$$t=0,\ 2,\ -2$$

t	x	y
0	0	-2
2	0	2
-2	0	2

When is the tangent horizontal?

$$\frac{dy}{dx} = 0$$

$$\frac{dy/dt}{dx/dt} = 0$$

$$\frac{dy}{dt} = 0$$

$$0 = \frac{dy}{dt} = 2t \qquad t = 0$$

vertical

$$\frac{dy/dt}{dx/dt} = \frac{*}{0}$$

$$\frac{dx}{dt} = 0$$

$$3t^2 - 4 = 0$$

$$t^2 = 4/3$$

$$t = \pm\sqrt{4/3} = \pm\frac{2}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}} < \sqrt{2}$$