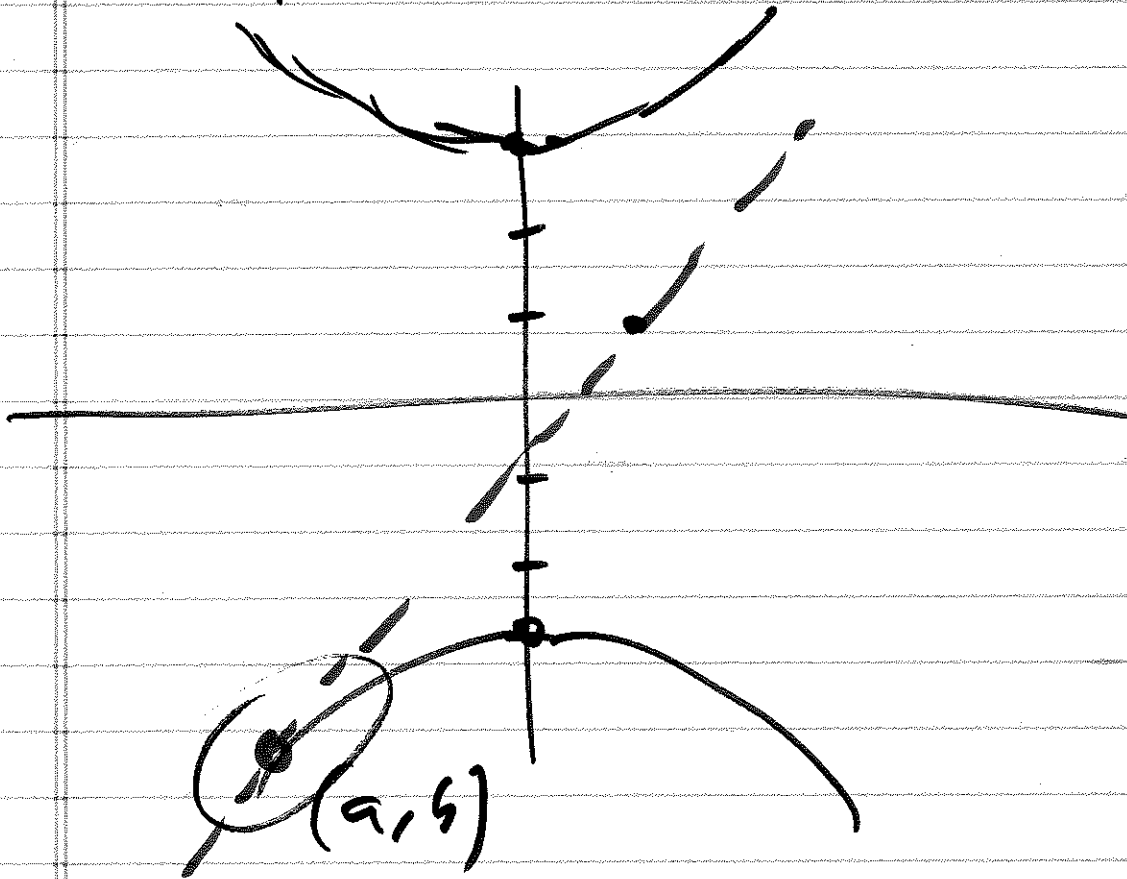


- Worksheet

- Implicit Diff

Ex:  $y^2 - x^2 = 9$        $(1, 1)$



Q: Find  $(a, b)$

so tangent at  
 $(a, b)$  passes thru  $(1, 1)$   
←

$$y - b = m(x - a)$$

$$(*) \quad b^2 - a^2 = 9$$

$$1 - b = m(1 - a)$$

differentiate implicitly

$$2y y' - 2x = 0$$

$$y' = \frac{x}{y} \Big|_{(a, b)} = \frac{a}{b}$$

$$b \cdot 1 - b = \frac{a}{b} (1 - a) \cdot b \quad (**)$$

$$b - b^2 = a(1 - a)$$

$$= a - a^2$$

$$b - a = b^2 - a^2 = 9 \quad (**)$$

$$\begin{cases} b - a = 9 \\ b^2 - a^2 = 9 \end{cases}$$

$$(b - a)(b + a) = 9$$

$$9$$

$$9(b + a) = 9$$

$$b - a = 9$$

$$b - 9 = a$$

$$+ \quad b + a = 1$$

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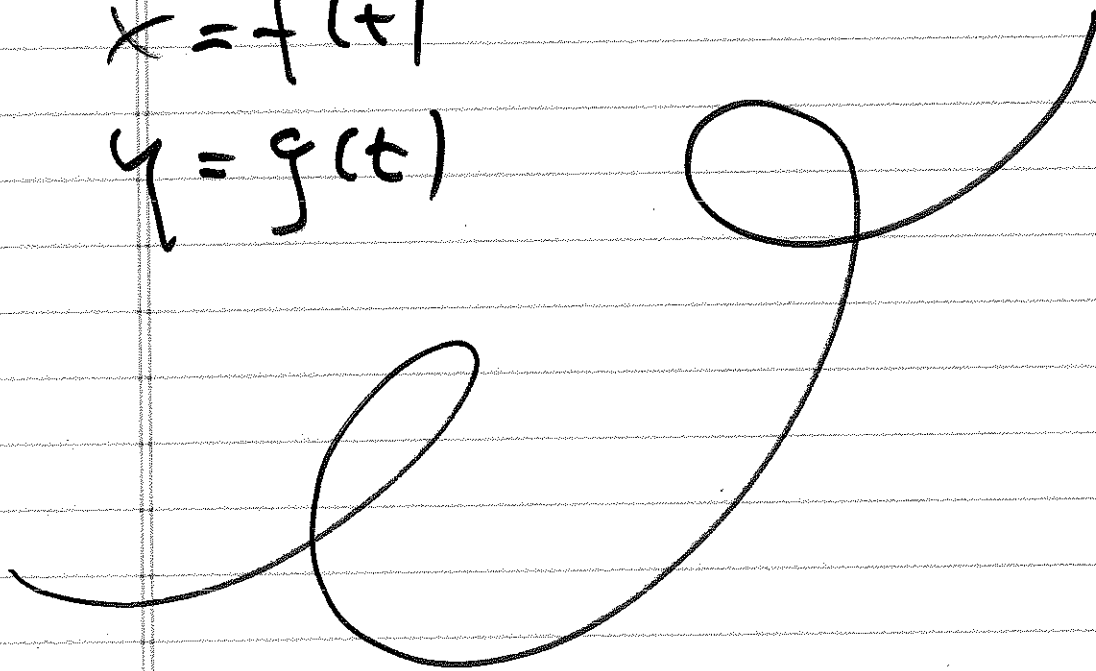
$$2b = 10$$

$$b = 5 \quad a = -4$$

# Parametric Equations

$$x = f(t)$$

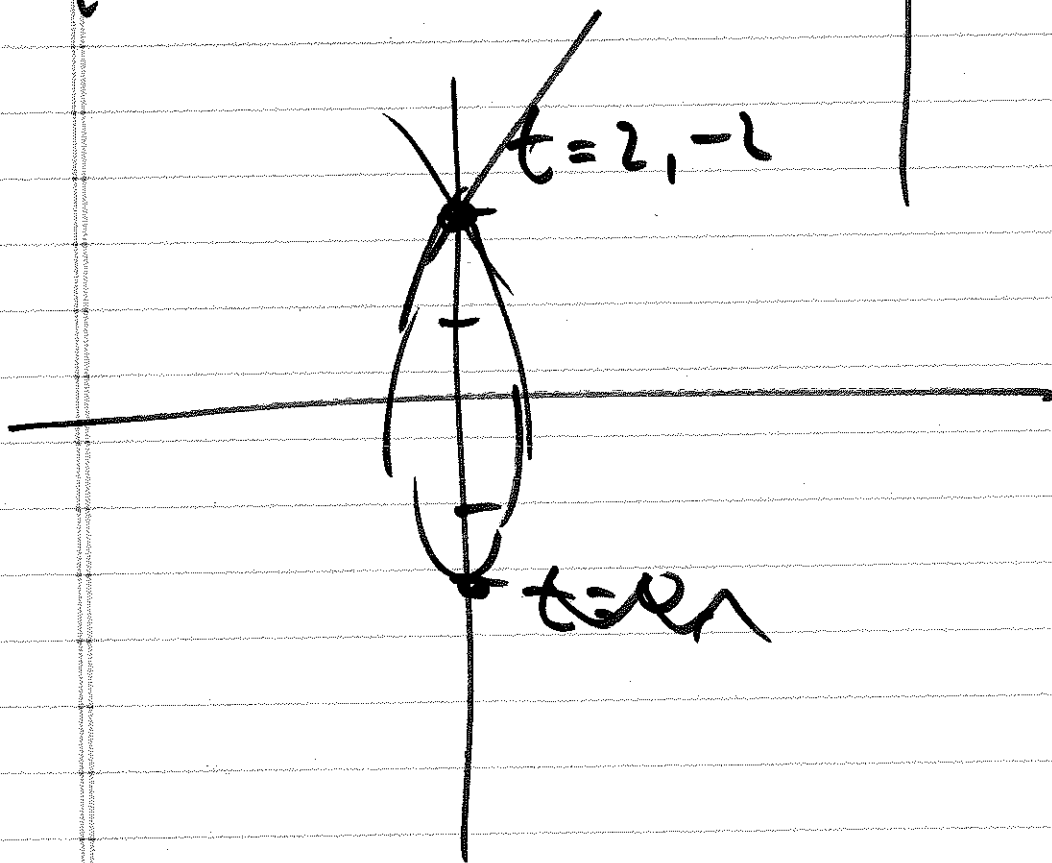
$$y = g(t)$$



~~x = x~~

$$\begin{cases} x = t^3 - 4t \\ y = t^2 - 2 \end{cases}$$

t	x	y
0	0	-2
-2	0	2
2	0	2



$$x = t(t^2 - 4) = t(t+2)(t-2)$$

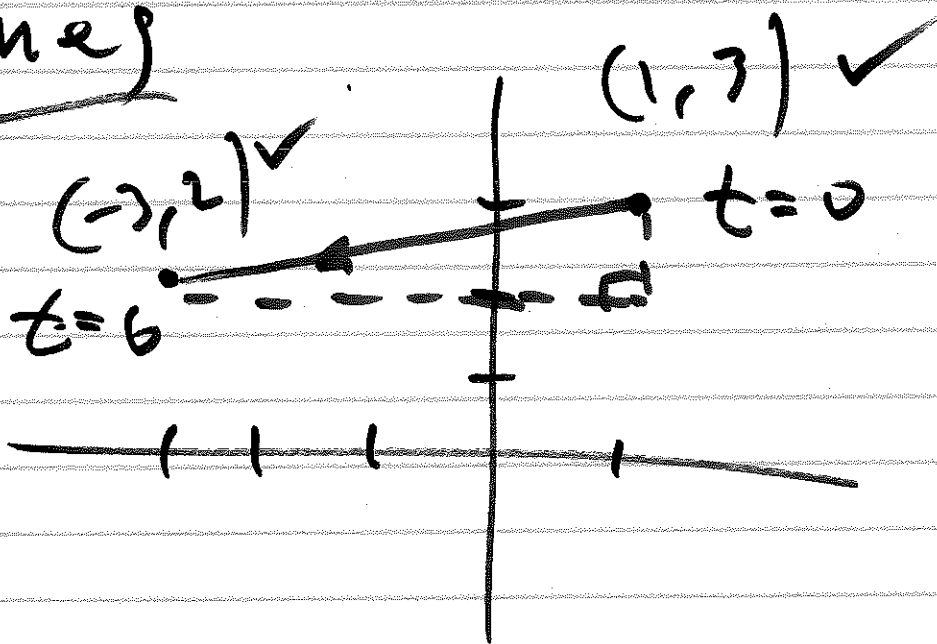
3 types

① lines

② projectiles

③ circles / ellipses

Lines



constant speed

$$x = at + b$$

$$y = ct + d$$

$$x(0) = a \cdot 0 + b = b = 1$$

$$y(0) = d = 3$$

$$a = \frac{\Delta x}{\Delta t} = v_x = \frac{-3 - 1}{6 - 0} = -\frac{2}{3}$$

$$c = \frac{\Delta y}{\Delta t} = v_y = \frac{2 - 3}{6 - 0} = -\frac{1}{6}$$

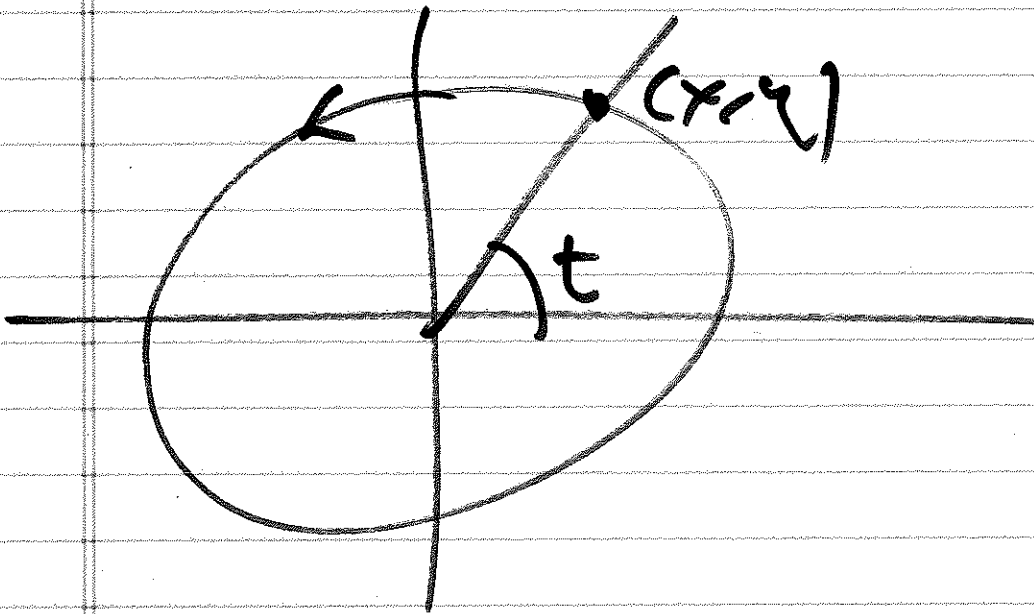
$$\begin{cases} x = -\frac{2}{3}t + 1 \\ y = -\frac{1}{6}t + 3 \end{cases}$$

$$\begin{aligned} x(6) &= -\frac{2}{3} \cdot 6 + 1 = -3 \quad \checkmark \\ y(6) &= -\frac{1}{6} \cdot 6 + 3 = 2 \end{aligned}$$



Circular

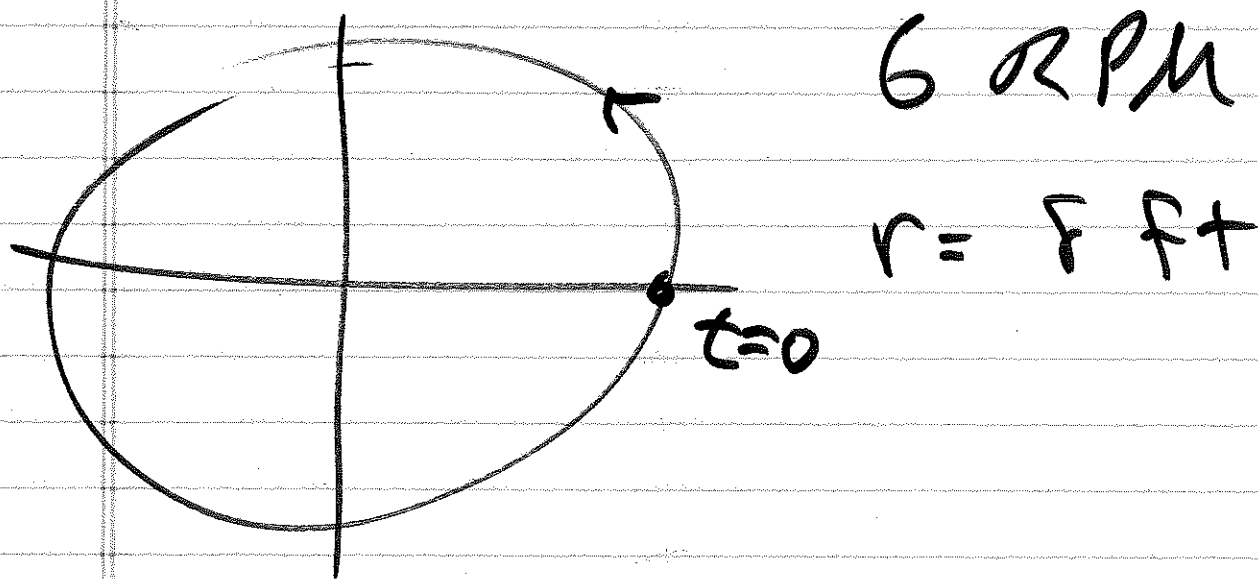
$$\begin{cases} y = \sin t \\ x = \cos t \end{cases}$$



$$x^2 + y^2 = (\cos t)^2 + (\sin t)^2 \\ = 1$$

eliminate parameter

Ex: Merry-go-round



$$\begin{cases} x = 8 \cos(12\pi t) \\ y = 8 \sin(12\pi t) \end{cases}$$

$$x = a + r \cos \theta$$

$$y = b + r \sin \theta$$

Center

$$\theta = \omega t + \theta_0$$

angular speed

initial angle

$$\omega \frac{\text{rad}}{\text{min}}$$

$$\frac{\Delta \theta}{\Delta t}$$

$$a, b = 0$$

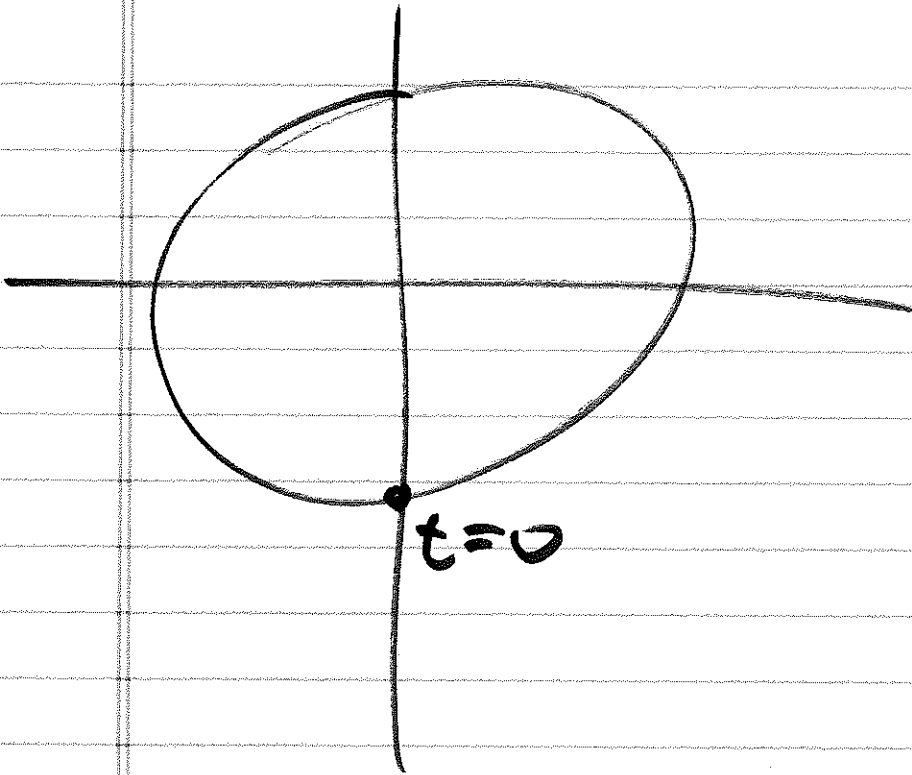
$$r = 8$$

$$\theta_0 = 0$$

$$\omega ? \quad 6 \frac{\text{rev}}{\text{min}}$$

$$6 \frac{\text{rev}}{\text{min}} \frac{2\pi \text{ rad}}{1 \text{ rev}}$$

$$\omega = 12\pi \text{ rad/min}$$



$$x = 8 \cos(12\pi t - \pi/2)$$

$$y = 5 \sin(12\pi t - \pi/2)$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

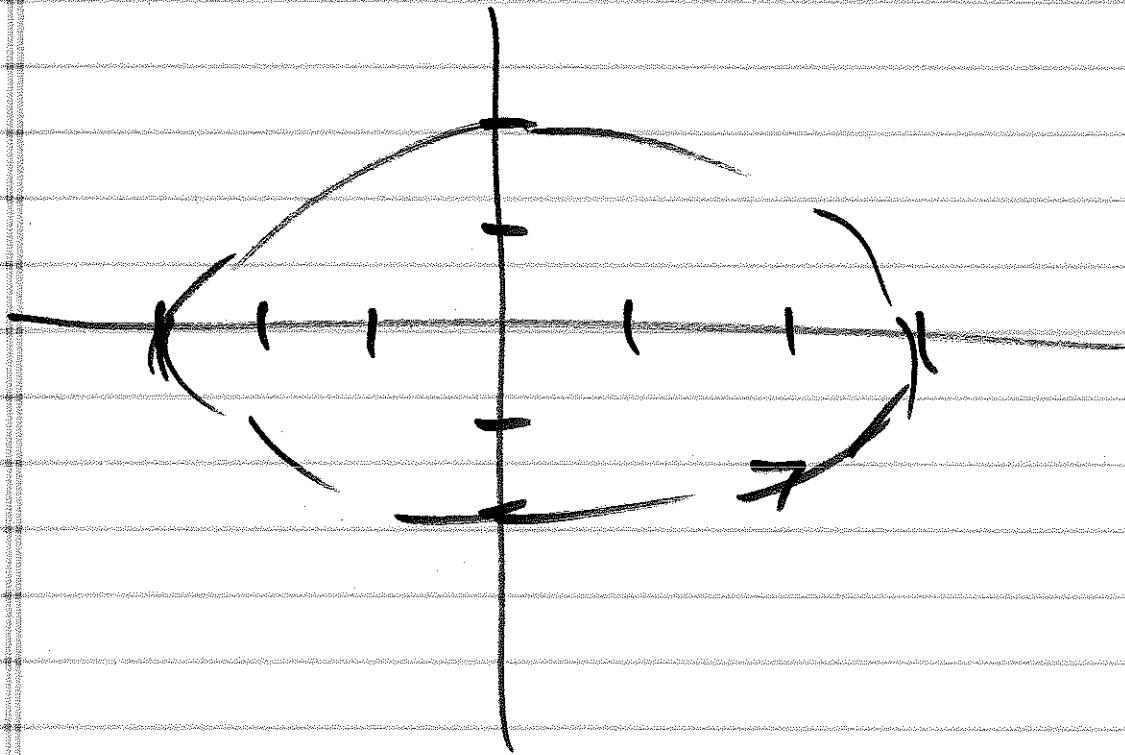
Ellipse

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

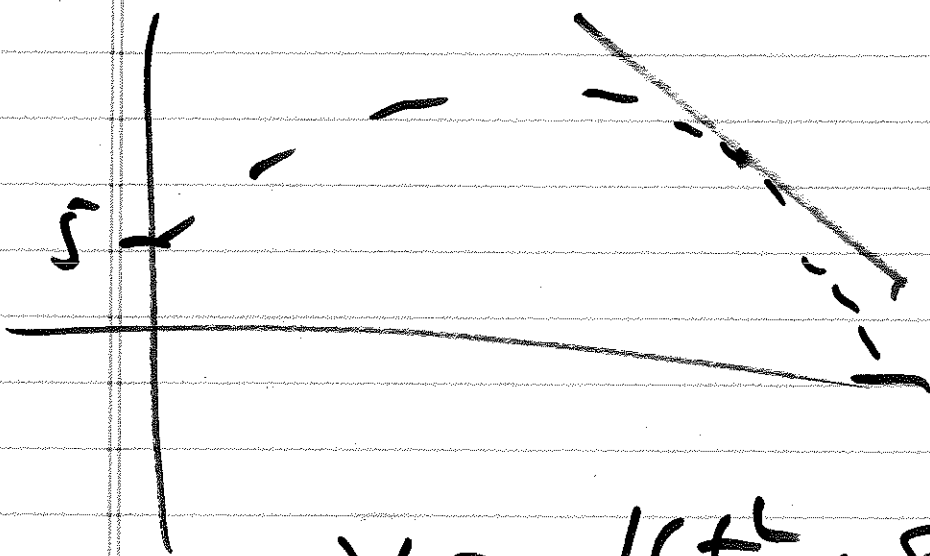
$$\cos t = \frac{x}{2}$$

$$\sin t = \frac{y}{3}$$

$$\begin{cases} x = 2 \cos t \\ y = 3 \sin t \end{cases}$$



projectile



$$y = -16t^2 + 8t + 5$$

$$x = 4t$$

ft, sec

$$x(0) = 0$$

$$y(0) = 5$$

$$V_x = \frac{dx}{dt} = 4 \text{ ft/sec} = \dot{x}$$

$$V_y = \frac{dy}{dt} = -32t + 8$$

$\ddot{x}$  max height?

$$\frac{dy}{dt} = 0$$

$$-32t + 8 = 0$$

$$t = 1/4 \text{ sec}$$

Trajectory?