

Implicit DFR:

Midterm:

write a note ✓  
give it to TA



Conchoid of Nicomedes

$$x^2 y^2 = (y+1)^2 (4-y^2)$$

$(\sqrt{2}, 1)$  is on the curve?

② tangent

$$y=1$$

$$x^2 = (1+1)^2 (4-1)$$

$$= 4 \cdot 3$$

$$= ~~12~~ 12 \quad \checkmark$$

$$\rightarrow 4$$

$$y-1 = m(x - \sqrt{12})$$

$$m = y' \quad \left( \frac{dy}{dx} \right)$$

$(\sqrt{12}, 1)$

$$\frac{d}{dx} x^2 \cdot y^2 = \frac{d}{dx} (y+1)^2 \cdot (4-y^2)$$

$$\begin{aligned} & 2xy^2 + x^2 \cdot 2y \cdot y' \\ & = 2(y+1) \cdot y' (4-y^2) \\ & \quad + (y+1)^2 \cdot -2y \cdot y' \end{aligned}$$

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$$\frac{d}{dx} (y+1) = y'$$

$$\frac{d}{dx} 4-y^2 = -2y \cdot y'$$

$$y-1 = \frac{-\sqrt{12}}{10} (x-\sqrt{12})$$

$$-\frac{\sqrt{3}}{5}$$

plug in now!

$$x = \sqrt{12}$$

$$y = 1$$

$$2x + 2x^2 y' = 2 \cdot 2 \cdot y' \cdot 3 \\ + 2 \cdot -2y'$$

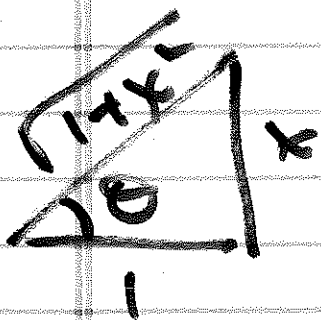
$$2x + 2x^2 y' = 12y' - 8y' \\ = 4y'$$

$$2\sqrt{12} + 2 \cdot 12 y' = 4y'$$

$$2\sqrt{12} = -20y'$$

$$-\frac{\sqrt{12}}{10} = y'$$

Ex:  $y = \tan^{-1} x$



$$\theta = y$$

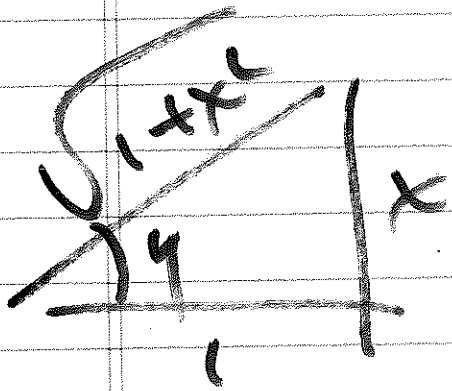
$$\begin{aligned} \tan(y) &= \tan(\tan^{-1} x) \\ &= x \end{aligned}$$

$$\tan y = x$$

$$\frac{d}{dx} \tan y = \frac{d}{dx} x$$

$$\sec^2 y \cdot y' = 1$$

$$y' = \frac{1}{\sec^2 y}$$



$$\sec y = \frac{\sqrt{1+x^2}}{1}$$

$$y' = \frac{1}{(\sqrt{1+x^2})^2}$$

$$= \frac{1}{1+x^2}$$

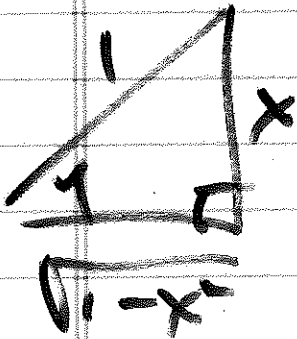
Ex:  $\frac{d}{dx} \sin^{-1} x$

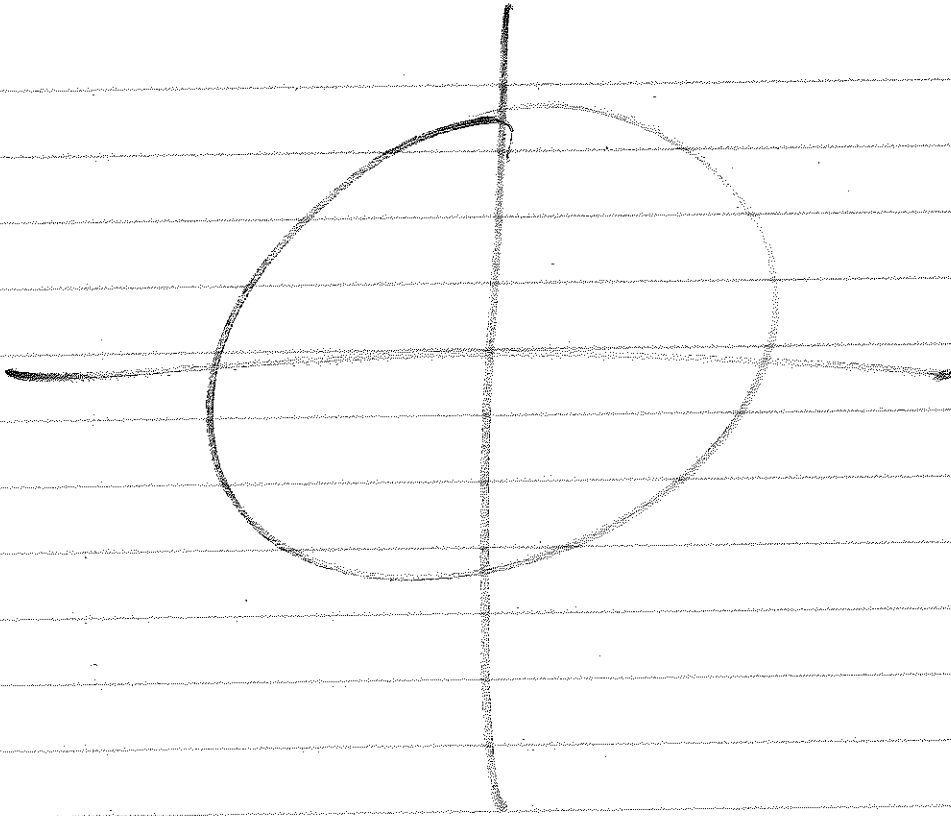
$$y = \sin^{-1} x$$

$$\sin y = x$$

$$\cos y \cdot y' = 1$$

$$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$





Ex:

$$y = \ln x$$

$$e^y = e^{\ln x}$$

The variables  $y$  and  $x$  in the equation above are circled.



$$\frac{d}{dx} e^y = \frac{d}{dx} x$$

$$e^y \cdot y' = 1$$

$$y' = \frac{1}{e^y} = \frac{1}{x}$$

~~4~~

$$\text{Eg: } 2x^2y + y^3 = 1 \quad (0, 1)$$

$$y'' \Big|_{(0,1)}$$

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$$4 + 3y'' = 0$$

$$y'' = -\frac{4}{3}$$

$$(1) \cancel{4y} + \cancel{2y^2} y' + 3y^2 y' = 0$$

$$(2) \quad 4y + \cancel{4y} y' + \cancel{2y^2} y'' + 6y y' \cdot y' + 3y^2 \cdot y'' = 0$$

plug in (0,1) to (2)

$$4 + \cancel{6y^2} y' + 3y'' = 0$$

plug (0,1) into (1)

$$3y' = 0 \quad y' = 0$$