Chain Rule

Ex: $y = \csc\left(\frac{x-1}{x+1}\right)$

$$\frac{dy}{dx} = -\csc\left(\frac{x-1}{x+1}\right)\cot\left(\frac{x-1}{x+1}\right) \cdot \frac{1\cdot(x+1) - (x-1)\cdot 1}{(x+1)^2}$$

$$\frac{2}{(x+1)^2}$$

Ej: $y = 7^x$ | $\ln e^x = x$, $e^{\ln x} = x$

$$\ln y = \ln 2^x$$

① $\ln AB = \ln A + \ln B$

② $\ln A/B = \ln A - \ln B$

③ $\ln A^B = B \ln A$

$$\ln y = \ln 7^x = x \ln 7$$

$$(1.000000001)^n$$

$$e^{\ln y} = e^{x \ln 7}$$

$$\boxed{y = e^{x \ln 7}}$$

$$\frac{dy}{dx} = e^{x \cdot \ln 7} \cdot 1 \cdot \ln 7$$

$$= 7^x \cdot \ln 7$$

$$\frac{d}{dx} a^x = a^x \cdot \ln a$$

Week 11 Sec

$$y = x^x$$

Ex: $f(x) = x^{7/5}$

$$[f(x)]^5 = x^7$$

$$\frac{d}{dx}[f(x)]^5 = \frac{d}{dx}x^7$$

$$5(f(x))^4 \cdot f'(x) = 7x^6$$

$$5(x^{7/5})^4 \cdot f'(x) = 7x^6$$

$$5 \cdot x^{28/5} \cdot f'(x) = 7x^6$$

$$f'(x) = \frac{7}{5} \frac{x^6}{x^{28/5}}$$

$$= \frac{7}{5} x^{6-28/5} = \frac{7}{5} x^{2/5}$$

$$6 - 28/5$$

$$\frac{30}{5} - \frac{28}{5} = \frac{2}{5}$$

Eg: $x^2+y^2=25$

tangent line at $(4,3)$

① use geometry

② write $y=f(x)$ explicit fcn

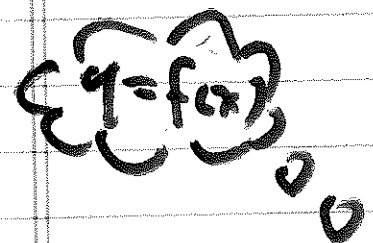
$y^2 = 25 - x^2$

$y = \pm\sqrt{25-x^2}$

at $(4,3)$

$y = \sqrt{25-x^2}$

③ implicit



$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}\,2$$

(think $\frac{d}{dx}(x^2+(f(x))^2) = \frac{d}{dx}\,2$)

$$\frac{d}{dx}(f(x))^2 = 2f(x)\cdot f'(x)$$

$$\frac{d}{dx}y^2 = 2y\cdot\frac{dy}{dx}$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}\Big|_{(4,3)}$$

$$= -\frac{4}{3}$$

Implicit diff

Ex: $x^3 + 2xy + y^2 = 0$

$$\frac{d}{dx}\left(x^3 + 2xy + y^2\right) = \frac{d}{dx}\,0$$

$$3x^2 + 2y + 2xy' + 2yy' = 0$$

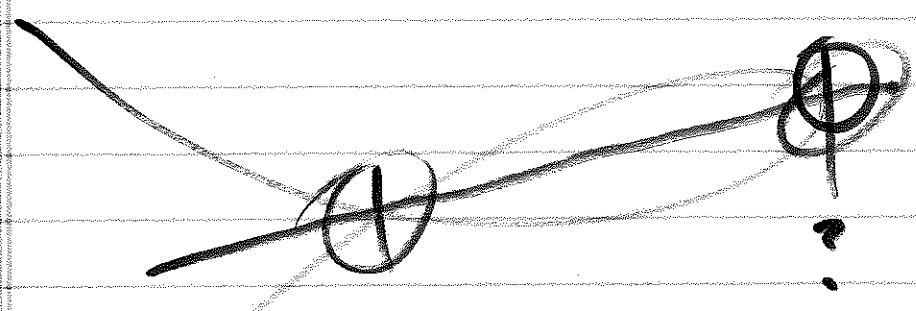
$$\frac{d}{dx}\left[\boxed{2x}\cdot\boxed{y}\right] = 2y + 2xy'$$

$$2xy' + 2yy' = -3x^2 - 2y$$

$$y'(2x + 2y) = \text{————}$$

$$y' = \frac{-3x^2 - 2y}{2x + 2y} \qquad \frac{0}{0} \; ?!$$

$x = -y$ at $(0,0)$



$x^3 + 2xy + y^2 = 0$

$y' = \frac{*}{0}$

$2x + 2y = 0$

$y = -x$ ✓

Intersection

elimination

$x^3 + 2x(-x) + (-x)^2 = 0$

$x^3 - 2x^2 + x^2 = 0$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

(0, 0) $\frac{0}{0} = y'$??

(1, -1) ✓ $y' = \frac{-3-2-1}{0}$

$= \frac{-1}{0}$ ✓

vertical