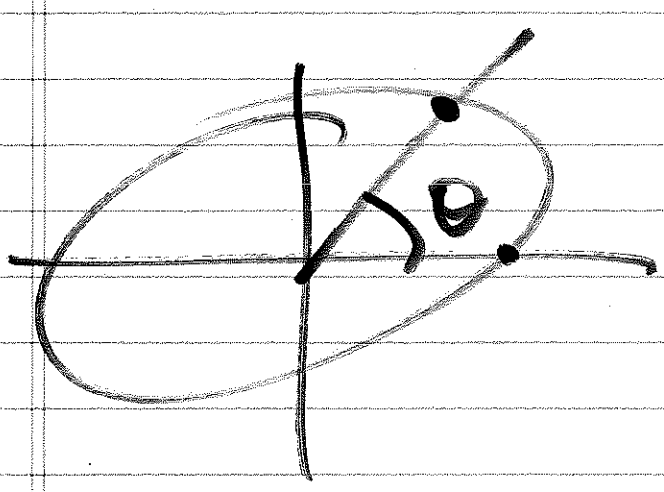
Trigonometric functions



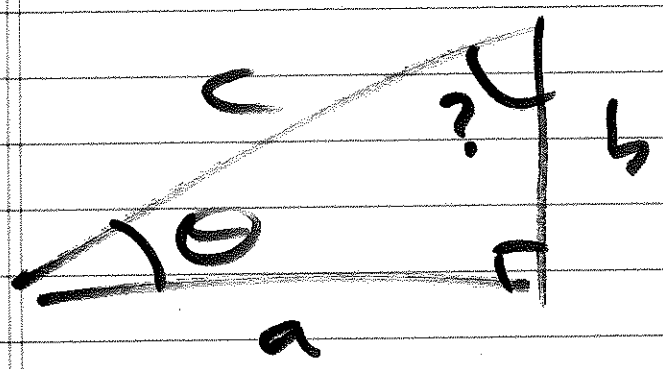
$x^2+y^2=1$

$(\cos\theta, \sin\theta)$

$\sin\theta \quad \cos\theta = 1$

$\sin^2 x + \cos^2 x = 1$

identities



$$\sin\theta = b/c$$

$$\cos\theta = a/c$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = b/a$$

$$\theta + \pi/2 + ? = \pi$$

$$? = \frac{\pi}{2} - \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \frac{a}{c}$$

$$= \cos\theta$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\pi} = -1$$

$$1 + e^{i\pi} = 0$$

$$\sin(A+B) = \sin A\cos B + \sin B\cos A$$

$$\frac{d}{dx}\sin x$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

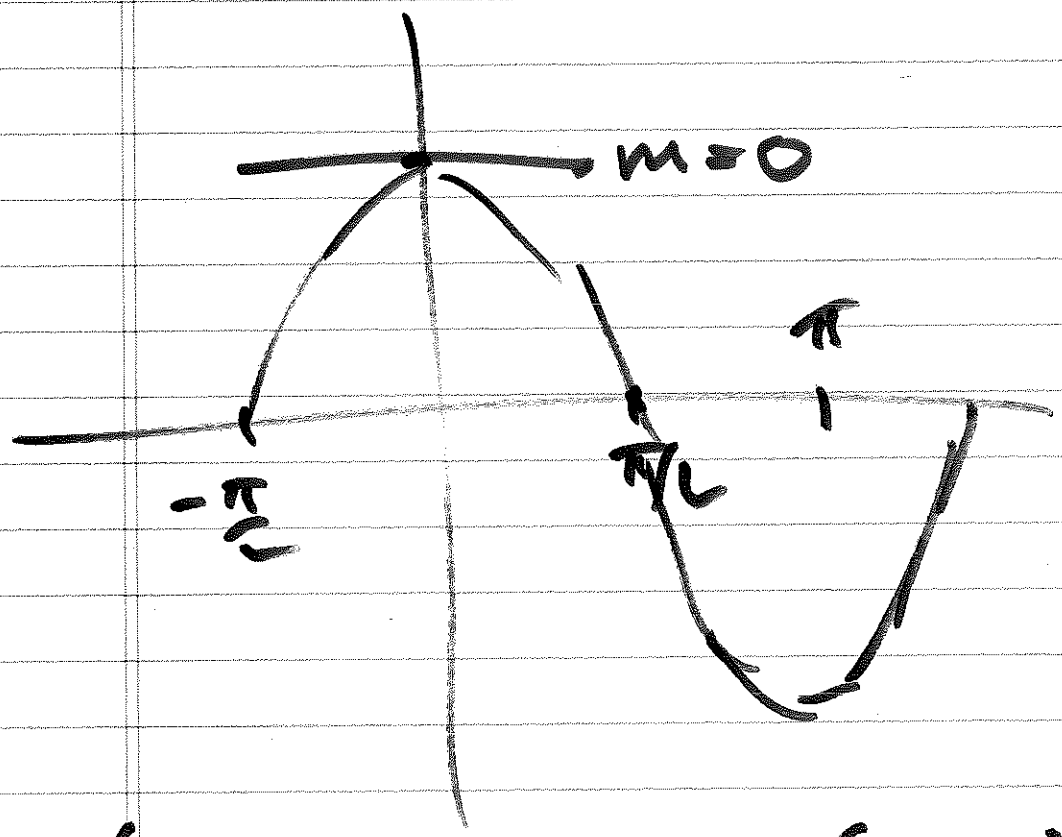
$$= \lim_{h\to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h - \sin x}{h} + \frac{\sin h \cos x}{h}$$

$$= \sin x \underbrace{\left[\lim_{h\to 0} \frac{\cos h - 1}{h}\right]}_{0} + \cos x \underbrace{\left[\lim_{h\to 0} \frac{\sin h}{h}\right]}_{1}$$

$$\frac{\cos h - 1}{h} \cdot \frac{\cos h + 1}{\cos h + 1}$$

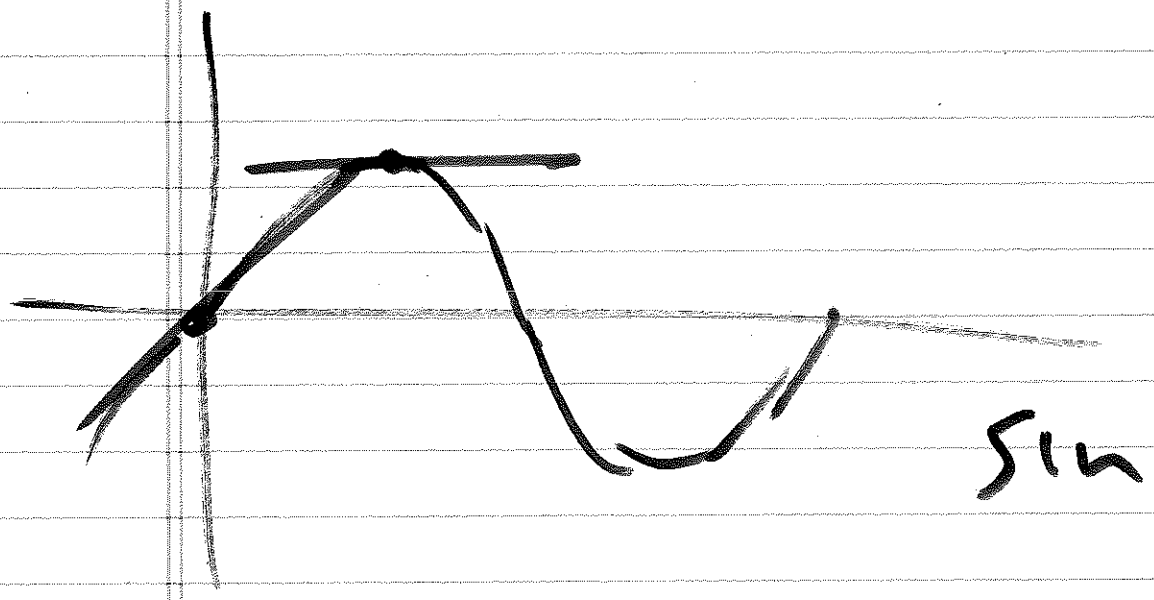
$$f(x) = \cos x$$

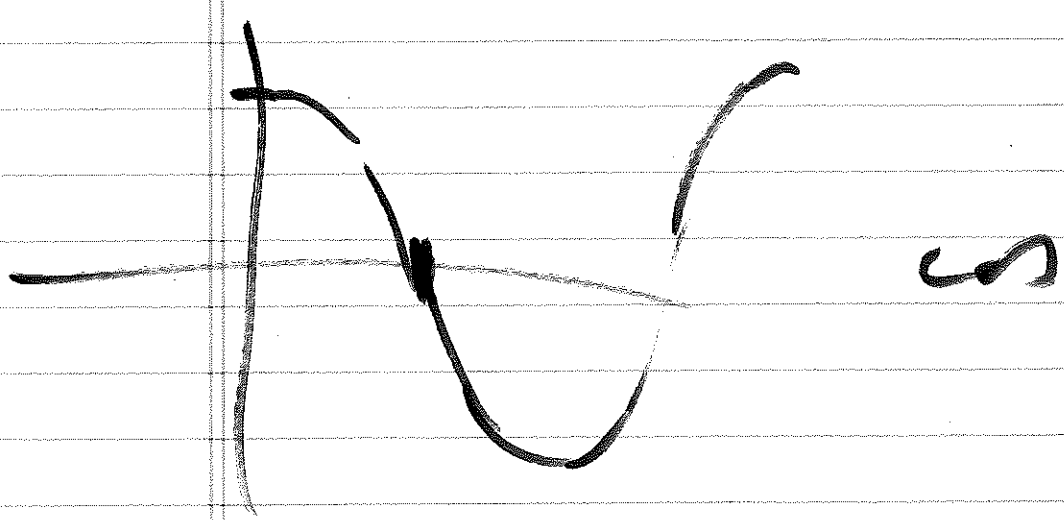


$$f'(0) = \lim_{h \to 0} \frac{\cos(0+h) - \cos 0}{h}$$

$$= 0$$

$$\frac{d}{dx} \sin x = \cos x$$





Chain Rule

$y = g(f(x))$

$\sqrt{1+x^2}$

Here $y = \sin(f(x))$

$\frac{dy}{dx} = \cos(f(x)) \cdot f'(x)$

If $f(x) = \frac{\pi}{2} - x$

$f'(x) = -1$

Eg: $\frac{d}{dx} x^2 \sin x$

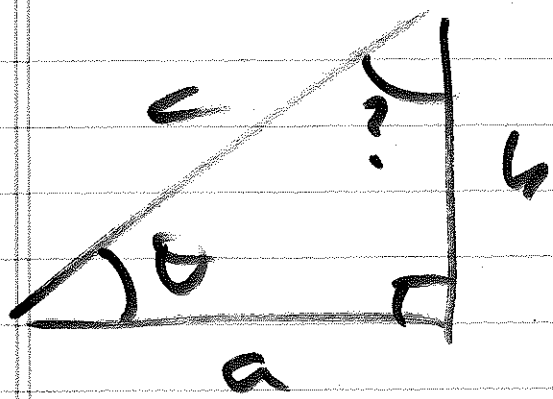
$= 2x \sin x + x^2 \cos x$

$\frac{d}{dx} \cos x$?

$= \frac{d}{dx} \sin\left(\frac{\pi}{2} - x\right)$

$= \cos\left(\frac{\pi}{2} - x\right) \cdot -1$

$= -\sin x$



$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{b}{c}$$

$$= \sin\theta$$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x}$$

$$\frac{\cos x \cdot \cos x - \sin x \cdot -\sin x}{\cos^2 x}$$

$$(\cos x)' = \cos^2 x$$

$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\left(\frac{1}{\cos x}\right)^2$$

Ex: $f(\theta) = \underset{u}{\theta} \cdot \underset{v}{e^{\theta}} \cdot \underset{w}{\sec\theta}$

$$f'(\theta) = 1 \cdot e^{\theta}\sec\theta$$

$$+ \theta \cdot e^{\theta}\sec\theta$$

$$+ \theta \cdot e^{\theta} \cdot \sec\theta \cdot \tan\theta$$

$$(uvw)'$$

$$(u \cdot (vw))'$$

$$u' \cdot vw + u \cdot (vw)'$$

$$u'vw + u(v'w + vw')$$

$$u'vw + uv'w + uvw'$$

$$\frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x}$$